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Bargaining Competition and Vertical Mergers

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Abstract

Vertical merger models are complex systems built on (i) a network of, e.g., upstream sellers and downstream retailers (ii) who bargain bilaterally in the presence of externalities (iii) created by competition between downstream retailers (iv) facing a consumer demand surface. We simulate the effects of vertical mergers in seven different models to identify which, and by how much, various assumptions about the nature of bargaining lead to anticompetitive outcomes. We find, among other things, that:

- Compared to derived demand models, bargaining models increase the scope for anticompetitive outcomes because they reduce upstream margins which reduces the benefit of merger, the elimination of double marginalization.
- *Nash-in-Nash* and *Nash-in-Shapley* can give opposing predictions about whether vertical mergers are anticompetitive.

Because the assumptions about the nature of bargaining can predetermine outcomes, it is critical to ensure that a model captures the significant features of pre-merger competition, and the loss of such competition following merger. However, this may be difficult to do because many of the assumptions about bargaining–critically, the alternatives to agreement–are unobservable.

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1 Introduction

Vertical merger models are built on: (i) a network of, e.g., upstream suppliers and downstream retailers; (ii) who bargain bilaterally in the presence of externalities; (iii) created by competition between downstream retailers; (iv) facing a consumer demand surface. Trying to understand how such a "complex system" (Wolfram, 2002) works is difficult. Indeed, in the 2016 AT&T-TimeWarner merger challenge, the first litigated vertical merger case in forty years, the judge called the government's model a "Rube Goldberg' machine"[1] before ruling for the defendants. The trial highlighted the uncertainty surrounding such "bargaining competition" i.e. how parties bargain in the presence of externalities created by competition. It has also motivated (or coincided with) recent academic interest in the topic (Crawford et al., 2018; Froeb et al., 2019; Rogerson, 2020; Sheu and Taragin, 2017; Yu and Waehrer, 2018). The trial lead to draft vertical merger guidelines by the US agencies (DOJ and FTC, 2020), after hearings that included seventy-four written comments from leading academics and practitioners (FTC, 2020).

Direct observation about how to characterize bargaining competition is difficult because the alternatives to agreement that determine the terms of agreement (Nash, 1950) are typically not observed. Even when we can observe them, Friedman (1953) reminds us that the important point is not whether the assumptions are descriptively realistic, for they never are, but "sufficiently good approximations for the purpose in hand."

Our purpose is vertical merger and to determine how well the assumptions work, we first have to understand what they do. In particular, we want to know how assumptions about (i) how parties bargain, (take-it-or-leave-it, Nash-in-Nash, or Nash-in-Shapley) and, (ii) over what (linear prices, two-part prices, or quantity), affect merger outcomes. We want to determine which assumptions matters, why they matter, but most importantly "how much" they matter so that we can answer the difficult question of model selection, "in a given or situation, which assumptions best capture the significant features of competition, and the loss of such competition following merger?" (Werden et al., 2004).

It is well-established that take-it-or-leave-it bargaining in the "derived demand" model, the oligopoly successor to the successive-monopoly model (Church, 2008), leads to price above that of monopoly (O'Brien and Shaffer, 1992; Moresi et al., 2007). As a consequence, the elimination of double marginalization (EDM) is big, which means that the scope for anticompetitive vertical mergers is small. In contrast, Bargaining models that split the gains from trade reduce the size of EDM typically increase the scope for anticompetitive outcomes (Sheu and Taragin, 2017).

The popular Nash-equilibrium-in-Nash-bargains (Nash-in-Nash) has the virtue that it provides easily computable outcomes for complicated bargaining environments which has made it an empirical "workhorse" (Collard-Wexler et al., 2019). However, this tractability comes at some cost. Nash-in-Nash outcomes depend on who earns operating profit, a violation of the Coase Theorem(Froeb et al., 2019;) and the Nash assumption that all other

[1] "Kabuki Dances or Rube Goldberg Machines? Vertical Analyses of Media Mergers," Competition Policy International (August 17, 2018), at https:// www.competitionpolicyinternational.com/kabuki-dances-or-rube-goldberg-machines-vertical-analyses-of-media-mergers/.

bargains are held constant implies a relatively competitive pre-merger equilibrium as parties agree on low wholesale prices to enable their downstream partners to price aggressively (Rey and Vergé, 2019), i.e. without much concern as to how such agreements may affect agreements with others. When such "competition" is eliminated by vertical merger, price can increase significantly (Froeb et al., 2019).

In bargaining models that allow re-negotiation or contingent contracts which recognize such bargaining externalities, profit is split according to the Shapley value (Stole and Zwiebel, 1996; Inderst and Wey, 2003). In these models, parties bargain over wholesale prices in anticipation of how a follow-on non-cooperative game determines profit. Non-cooperative outcomes determine total profit and the profit of various threat points which determine how profit is split. (de Fontenay and Gans, 2014), Yu and Waehrer (2018) and Froeb et al. (2020) have used variants of this approach to characterize vertical mergers, sometimes called "Nash-in-Shapley" or "Nash-in-Nash with Recursive Threat Points."

This chapter provides a comparison of these two classes of bargaining models. We calibrate the models to a single monopoly equilibrium, and then simulate the effects of vertical mergers in various industry settings. This allows us to attribute the differences in outcomes to the different assumptions (how parties bargain and over what) of the models, allowing us to show what matters, why it matters, but also how much it matters.

In all the characterizations, vertical mergers give the merged firm a better outside option, resulting in a bigger profit share. We show that Nash-in-Shapley and Nash-in-Nash can give opposing merger predictions, but in the important case of bargaining over linear wholesale prices, the differences between Nash-in-Shapley and Nash-in-Nash quantitatively shrink because there is only one instrument (linear wholesale price) to both increase industry profit and split it up.

2 Vertical Merger Models

In this section, we describe the various elements of vertical merger models: (i) a network of upstream suppliers and downstream retailers (ii) who bargain bilaterally in the presence of externalities (iii) created by competition (iv) over consumer demand. We then describe the seven models and two benchmarks (Competition and Monopoly), which are summarized in Tables 1, 2 and 3 along with results of the computational experiments.

Network of Bilateral Trading Relationships

We consider two premerger industry structures:

- The 1×2 industry structure of one upstream firm, designated A, supplying two downstream firms, designated 1 and 2, with downstream firms competing for final consumers.

- The 2×1 industry structure by two upstream firms, designated A and B, supplying one downstream firm, designated 1, with the downstream firm selling two products to final consumers.

We imagine the upstream firm(s) as producing a product at some cost, agreeing to transfer product to the downstream firm(s) which have additional costs in selling to final to consumers.

For each industry structure we consider a vertical merger between A and 1. Before merger, each firm acts to maximize its own final profit. After a merger, the merged firm acts with respect to the merged firm's total final profit.

Forms of Agreement

We consider agreements of one of three possible forms: linear (one-part) pricing, assigning a marginal wholesale price; two-part pricing, specifying a marginal wholesale price and a fixed fee; and a specified quantity at a fixed price. For one- or two-part pricing, we assume that marginal wholesale price determines Nash equilibrium consumer prices, and thus product demands, and that everyone knows demand. For specified quantity, we assume that consumer prices are set to sell the specified quantities (no waste). For most cases, specifying quantity is equivalent to two-part pricing. The exception is *Nash-in-Nash*, which assumes others agreements are held constant in the alternatives to each agreement, so that the form of agreement matters.

Bargaining Models: Derived Demand, *Nash-in-Nash*, *Nash-in-Shapley*

In order to evaluate agreements between upstream and downstream firms, we need to consider what happens when agreements fail. We assume that when an upstream firm fails to agree with a downstream firm, that product is unavailable to the final consumer. For example, if A and 1 fail to agree, then the product A is not available through retailer 1.

For the two different market structures we consider the following sets of agreements:

- 1×2:
  - No agreements: $\{\emptyset\}$
  - A and 1 agree: $\{A1\}$
  - A and 2 agree: $\{A2\}$
  - Both agreements: $\{A1, A2\}$
- 2×1:
  - No agreements: $\{\emptyset\}$
  - A and 1 agree: $\{A1\}$

- B and 1 agree: $\{B1\}$
- Both agreements: $\{A1, B1\}$

If no agreements are reached $\{\emptyset\}$ and no products sold, we assume zero profit for all parties. In the post-merger equilibrium, we assume that the merged parties, e.g., A and 1, are automatically in agreement, so that the terms of agreement are irrelevant to the profit calculus of the merged firm.

We consider outcomes of agreements, but not the process of arriving at agreements. In the *Derived Demand* model, we imagine that the upstream firm simply dictates terms. For this specification, we do not consider the two-part pricing because the upstream firm would set marginal wholesale price to realize a desired downstream price and a fixed fee to extract all profit from the downstream firm, an unrealistic scenario. Instead we consider only the case of linear pricing dictated by the upstream firm(s) maximizing their own profit(s), resulting in the familiar double marginalization.

In other cases, we will assume that each agreement negotiated results from a Nash bargaining solution with respect to the parties' total profits over some disagreement point. For two-part pricing, solutions amount to firms maximizing their combined total profit and setting the fixed fee to split equally all profit over the threat point.

For linear, or one-part pricing, the firm(s) sets marginal wholesale price(s) so as to maximize a product of the form $(\pi_A - \pi_A^0)(\pi_1 - \pi_1^0)$, although it is possible to generalize this to consider the effect of differing bargaining powers, as in Crawford et al. (2018). For *Nash-in-Nash*, we assume the threat point is given by the profits determined in the case when all other agreements held fixed, e.g., Sheu and Taragin (2017). For *Nash-in-Shapley*, we assume the threat point is determined by profits with all other agreements adjusted for the new set of agreements, each of these determined recursively from cases with fewer agreements, e.g., (Froeb et al., 2020; Yu and Waehrer, 2018)

Downstream Demand: Rectangular Logit

In this section we introduce a demand system that can accommodate the various agreements (threat points) described above that can be reached by the upstream and downstream firms. We assume that each product sold through each downstream firm is a separate choice in a (nested) logit demand including a "no purchase" or outside option. For example, in the 1×2 case, if both agreements make, $\{A1, A2\}$, then consumers face a choice of product from A sold through 1, denoted A@1, or else the same product sold through 2, denoted A@2; and similarly in the the 2×1 case. When only one agreement is reached, consumers are reduced to a single option and the demand model is suitably adjusted. We assume a (nested) logit demand model, so eliminating one product corresponds to a limiting case where the price of that product goes to infinity. Logit demand makes it a simple matter to determine a change in consumer surplus between pre- and post-merger outcomes.

When there are a number of simultaneous agreements, understanding how the profit available in one agreement is affected by the presence of other agreements is critical, as these reflect the externalities that must be accounted for in bargaining over a given agreement. To illustrate, we take the example of agreements that allow for competing products to be sold to consumers. The sales of a product under one agreement will be negatively affected by a second agreement, whether it is the same product sold through an alternative retailer (the $1{\times}2$ case) or a different product sold through the same retailer (the $2{\times}1$ case). For substitute products, the other agreement represents a negative externality; and for complements, a positive one. The nested logit demand system all good are substitutes, which imply negative externalities caused by downstream product market competition.

Nested Rectangular Logit Demand

In this sub section, we characterize nested logit demand in terms of Kendall's τ (rank-order correlation) instead of the traditional nest strength parameter, $\theta = 1/(1-\tau)$. For those familiar with nested logit demand, or uninterested the derivation, skip to the last paragraph of this subsection to familiarize yourself with the notation.

Suppose there are n inside products, indexed 1 to n, together with an outside, no purchase, alternative indexed as 0. In our two industry structures, n will be at most 2, with products $A@1$ and $A@2$ in the 1×2 case or $A@1$ and $B@1$ in the 2×1 case. Let p_i be the price of the i-th inside goods, for each i, fixing $p_0 = 0$. Suppose that a consumer sees these products and prices and chooses one, with some total number of choices per specified time period, allowing for some scaling. If $(V_0, V_1, \ldots, V_n)$ is the $n+1$-tuple of values of a random consumer (nominated in the same units as prices), we suppose that the consumer will choose alternative i when, for all $j \neq i$, $V_i - p_i > V_j - p_j$ (ignoring possible ties). Let $X_i = V_i - p_i$ be the net value for alternative i, then the total demand for alternative i is $q_i = M \Pr(X_i > X_j, \text{all } j \neq i)$ where M is the total number of consumer choices (during some period). The outside quantity q_0 represents those consumers not choosing any of the inside products, usually not observable.

It is convenient to take the V_i to have marginal distributions that are extreme value with the same scale parameter λ and various location parameters η_i. Then the marginal distribution of X_i is also extreme value with scale parameter λ and location parameters $\eta_i^\dagger = \eta_i - p_i$, that is, with cumulative distribution function

$$F_i(t) = \Pr(X_i \leq t) = \exp\left(-\exp\left(-\frac{t-\eta_i^\dagger}{\lambda}\right)\right)$$

In fact, these distributions are power-related (Froeb et al., 2001). Anticipating the application below, write $F_i(t) = (F_{\max}(t))^{s_i^{1/\theta}}$, for a parameter $\theta \geq 1$, where

$$F_{\max}(t) = \exp\left(-\exp\left(-\frac{t-\eta_{\max}^\dagger}{\lambda}\right)\right)$$

is the extreme value distribution function with scale parameter λ and location parameter

$\eta^\dagger_{\max}$, where $s_i = \exp(\theta\eta^\dagger_i/\lambda)/\exp(\theta\eta^\dagger_{\max}/\lambda)$ and $\eta^\dagger_{\max}$ is taken so $\sum_i s_i = 1$, i.e.,

$$\eta^\dagger_{\max} = \frac{\lambda}{\theta}\log\left(\sum_{i=1}^{n}\exp\left(\frac{\theta\eta^\dagger_i}{\lambda}\right)\right)$$

A flat logit demand model results if the V_i (so X_i) are taken as independent, but a more general model is only slightly more complex. A Gumbel copula combines nicely with power-related distributed marginals to give a model with V_i correlated, and these can be simply combined in nests of smaller nests of increasing strengths. For our purposes it suffices to imagine X_0 is independent of $X_1 \ldots X_n$, but take these inside goods as a nest in a nested logit demand model. The Gumbel copula is the Archimedean copula given by generator $\psi_\theta(t) = (-\log(t))\theta$ for parameter $\theta \geq 1$ reflecting the strength of the correlation, with $\theta = 1$ the limiting case of independence. For a nest of n variables, the copula is $C(u_1,\ldots,u_n;\theta) = \psi_\theta^{-1}(\psi_\theta(u_1) + \ldots + \psi_\theta(u_n))$, where $\psi_\theta^{-1}(t) = \exp(-t^{1/\theta})$, That is, C is a joint cumulative distribution function with uniform marginals and the joint distribution function of the inside X_i is taken to be

$$\begin{aligned}
F_{1\ldots n}(t_1,\ldots,t_n) &= \Pr(X_i \leq t_i, \text{for all } i > 0) \\
&= C(F_1(t_1),\ldots,F_n(t_n);\theta) \\
&= C(F_{\max}(t_1)^{s_1/\theta},\ldots,F_{\max}(t_n)^{s_n/\theta};\theta) \\
&= \psi_\theta^{-1}(\psi_\theta(F_{\max}(t_1)^{s_1^{1/\theta}}) + \ldots + \psi_\theta(F_{\max}(t_n)^{s_n^{1/\theta}})) \\
&= \psi_\theta^{-1}(s_1\psi_\theta(F_{\max}(t_1)) + \ldots + s_n\psi_\theta(F_{\max}(t_n))) \quad \text{and so} \\
F_{1\ldots n}(t,\ldots,t) &= \Pr(\max_{i>0} X_i \leq t) \\
&= \psi_\theta^{-1}((s_1 + \ldots + s_n)\psi_\theta(F_{\max}(t))) \\
&= F_{\max}(t)
\end{aligned}$$

In general, the distribution of the maximum of n random variables having joint distribution given be the Gumbel copula applied to power-related marginal distributions is also power-related, and taking this maximum distribution as base distribution the marginal distributions are $F_{\max}(t_i)^{s_i/\theta}$ where the s_i are shares that sum to unity. Moreover (under mild conditions),

$$
\begin{aligned}
\Pr(X_i > X_j, \text{all } 0 < j \neq i) &= \int_{\substack{t_i > t_j,\\ \text{all } j \neq i}} dF_{1\ldots n}(t_1, \ldots, t_n) \\
&= \int_{t_i} \frac{\partial}{\partial t_i} F_{1\ldots n}(t_1, \ldots, t_n)\bigg|_{t_1 = t_i, \ldots, t_n = t_i} dt_i \\
&= \int_{t_i} (\psi_\theta^{-1})'(s_1 \psi_\theta(F_{\max}(t_i)) + \ldots + s_n \psi_\theta(F_{\max}(t_i))) \\
&\qquad\qquad \cdot s_i \psi_\theta'(F_{\max}(t_i)) F_{\max}'(t_i)\, dt_i \\
&= s_i \int_{t_i} \frac{\partial}{\partial t_i} (\psi_\theta^{-1}(\psi_\theta(F_{\max}(t_i))))\, dt_i \\
&= s_i
\end{aligned}
$$

so the s_i reflect the probability that X_i is the maximum of $X_1, \ldots, X_n$. In fact, the distribution of the maximum of $X_1, \ldots, X_n$ is independent of the identity X_i that realizes that maximum.

$$
\begin{aligned}
\Pr(X_i \leq t | X_i > X_j, \text{all } 0 < j \neq i) &= \frac{1}{s_i} \Pr(t \geq X_i > X_j, \text{all } 0 < j \neq i) \\
&= \frac{1}{s_i} \int_{\substack{t \geq t_i > t_j,\\ \text{all } j \neq i}} dF_{1\ldots n}(t_1, \ldots, t_n) \\
&= \frac{1}{s_i} s_i \int_{t \geq t_i} \frac{\partial}{\partial t_i} (\psi_\theta^{-1}(\psi_\theta(F_{\max}(t_i))))\, dt_i \\
&= F_{\max}(t)
\end{aligned}
$$

Taking X_0 independent of the inside X_i combines X_0 with the maximum of the X_i in an outside nest with $\theta = 1$. The maximum of all X_i is thus extreme value distributed with scale parameter λ and location parameter

$$
\eta^\dagger_{\text{max all}} = \lambda \log \left(\exp\left(\frac{\eta^\dagger_0}{\lambda} \right) + \exp\left(\frac{\eta^\dagger_{\max}}{\lambda} \right) \right)
$$

The probability that X_0 is greater than any other X_i is thus

$$
\begin{aligned}
\pi_0 = \Pr(X_0 > X_i, \text{all } i > 0) &= \frac{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_{\text{max all}}}{\lambda}\right)} \\
&= \frac{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right) + \exp\left(\frac{\eta^\dagger_{\max}}{\lambda}\right)}
\end{aligned}
$$

The probability that X_i is greater than any X_j, $j \neq i$, is the probability that X_i is the maximum of X_j, $j > 0$, times the probability that the maximum of $X_1, \ldots, X_n$ exceeds X_0,

$$\pi_i = \Pr(X_i > X_j, \text{all } j \neq i) = s_i \cdot \frac{\exp\left(\frac{\eta^\dagger_{\max}}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_{\max\text{ all}}}{\lambda}\right)}$$

$$= s_i \cdot \frac{\exp\left(\frac{\eta^\dagger_{\max}}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right) + \exp\left(\frac{\eta^\dagger_{\max}}{\lambda}\right)}$$

Adding a constant to all of the η_i adjusts $\eta^\dagger_{\max}$ and $\eta^\dagger_{\max\text{ all}}$ by the same constant and hence leaves all of the choice probabilities π_i unchanged. We conventionally take $\eta_0 = 0$. The expected value of the maximum $X_i^\dagger$, the expected difference between the value of the product chosen by a random consumer and the price paid for that choice, is not determined without reference to an actual η_i, but the change in this quantity between two prices represents the change in consumer surplus in this model.

Sampling from this joint distribution of consumer values is not trivial when $\theta > 1$. A method for sampling from the Gumbel copula follows from work of Marshall and Olkin (1967). Sample V from the type 1 stable distribution with stability parameter $\alpha = 1/\theta$, skewness parameter $\beta = 1$, scale parameter $\sigma = \cos(\pi/2/\theta)^\theta$ and location parameter $\mu = 0$. Take W_i independent uniform $[0, 1]$. Then $U_i = \phi_\theta^{-1}(-\log(W_i)/V)$ are jointly distributed with distribution function $C(u_1, \ldots, u_n; \theta)$. From there we can take $X_i = F_i^{-1}(U_i)$ and $V_i = X_i + p_i$, with V_0 taken independent extreme value with scale parameter λ and location parameter $\eta_0 = 0$.

The correlation between variates defined by a copula is not independent of the marginal distributions but the Kendall rank correlation coefficient τ are, as they depend on rank orderings only). For two-particular products, if p is the probability that between two random consumers the one that values one product more highly is also the one that values the other product more highly, then $\tau = 2p - 1$. For the Gumbel copula, $\tau = 1 - 1/\theta$ is between 0 (independent) and 1 (in the limit). Put otherwise, for a specified Kendall $\tau \in [0, 1)$ we may take $\theta = 1/(1 - \tau)$ as nest parameter.

Summarizing, for a nested logit model with a nest around inside products having $\tau \in [0, 1)$, so nest parameter $\theta = 1/(1 - \tau)$, the demand for an inside product $i > 0$, and the total consumer surplus up to a constant, is given by,

$$q_i = M\frac{\exp\left(\frac{\eta_i - p_i}{\lambda(1-\tau)}\right)}{S + S^\tau}, \quad q_0 = M\frac{S^\tau}{S + S^\tau} \quad \text{where } S = \sum_{i=1}^{n} \exp\left(\frac{\eta_i - p_i}{\lambda(1-\tau)}\right)$$

$$CS = M\eta^\dagger_{\max\text{ all}} = M\lambda \log\left(1 + S^{1-\tau}\right)$$

where S=quantity share of the inside goods and CS is consumer surplus.

Benchmarks: Competition and Monopoly

We consider two benchmarks: competition and monopoly.

- Competition
  - In the 1×2 case, downstream firms acquire product at the upstream firm's marginal cost with downstream competition for consumers resulting in Nash equilibrium pricing.
  - In the 2×1 case, the two upstream firms compete for final consumers through a "transparent" downstream sector, with upstream wholesale prices reflected fully in downstream prices. (Froeb et al., 2017).
- Monopoly: In both the 1×2 and 2×1 cases, prices to consumers are set to maximize the total profit of all firms. When only one of the two products are taken as available to consumers, we consider for comparison a monopoly for only that product.

In every case, assuming two agreements are reached, we imagine a downstream consumer demand for two products; in the 1×2 case, a choice of the product from A and sold through 1, denoted $A@1$, or else the same product sold through 2, denoted $A@2$; in the 2×1 case, a choice of the product from A or else the product from B sold through 1 in either case, denoted $A@1$ and $B@1$ respectively. When only one agreement is reached, consumers are reduced to a single choice and the demand model is suitably adjusted.

3 Calibration

In this section we explain the calibration of the models discussed in section 2. We fix the scaling parameter (λ), initial prices ($p_{A@1}, p_{A@2}$ in the 1×2 setting and $p_{A@1}, p_{B@1}$ in the 2×1 setting), initial quantities of the inside goods ($q_{A@1}, q_{A@2}$ in the 1×2 setting and $q_{A@1}, q_{B@1}$ in the 2×1 setting) and the nest strength parameter (τ). We are ultimately interested in how the substitutability of the inside goods with the outside good affects the predictions of the system of models. As such, we exogenously increase the quantity of the outside good, from 10% to 210% of the sum of the inside goods by one percentage point at a time. Because the higher outside good quantity means that the total market size increases, aggregate elasticity in the market becomes more elastic. This provides us with a list of parameters: a varying outside good quantity with corresponding varying market size and aggregate elasticity, each with the same fixed parameters as stated above.

The above parameters are used to determine the location parameters (η_i) of the logit demand function:

$$\log\left(\frac{q_i}{\sum_{j=1}^{n} q_j}\right) = \frac{\eta_i - p_i}{\lambda(1-\tau)} - \log(S)$$

so

$$\eta_i = p_i + \lambda(1-\tau)\left(\log\left(\frac{q_i}{\sum_{j=1}^{n} q_j}\right) + \log(S)\right)$$

with

$$S = \left(\frac{q_0}{\sum_{i=1}^{n} q_i}\right)^{1/(\tau-1)}$$

Given these set of parameters, we can calibrate the demand model to prices and elasticities appropriate for the monopolist. We assume in the 1×2 and 2×1 case that the upstream firm(s) have zero marginal cost and that the marginal cost of the downstream firm(s) are inferred from monopoly assuming that the above prices are optimal. Specifically, for two products with total marginal costs mc_{tot1}, mc_{tot2}, the monopolist maximizes

$$\text{profit}_M = (p_1 - mc_{tot1})q_1 + (p_2 - mc_{tot2})q_2$$

so chooses prices satisfying first-order conditions

$$\begin{aligned}
0 &= q_1 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_1} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_1} \\
0 &= q_2 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_2} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_2}
\end{aligned}$$

a system of two linear equations easily solved for the total marginal costs. For the nested logit model there are certain simple relations. Substituting the derivative formulas,

$$\begin{aligned}
0 &= q_1 + (p_1 - mc_{tot1})\left(q_1 + q_1 s_1 \frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_1 s_2 \frac{f'(S)}{f(S)}\right) \\
0 &= q_2 + (p_1 - mc_{tot1})\left(q_2 s_1 \frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_2 + q_2 s_2 \frac{f'(S)}{f(S)}\right)
\end{aligned}$$

dividing by quantities and subtracting shows $p_1 - mc_{tot1} = p_2 - mc_{tot2}$, i.e., any difference in pricing for the monopolist is due to differences in total marginal cost.

The units on prices will not change the results of our calculations, so we may as well take the quantity weighted average price to be say $\bar{p} = 1$. The units on quantity similarly will not matter, so we may set the total quantity of inside products to say $q_{tot} = 100$. We further assume that the prices, quantities and marginal cost of the two products are equal (i.e. they are balanced).

This list of parameters: the initial prices, inside quantities, outside quantities, nest parameter, aggregate elasticity, scaling parameter and location parameters along with the conventions for $\bar{p} = 1$, $q_{tot} = 100$ and marginal costs inferred from monopoly pricing are enough to calibrate the demand model. These controls determine all of the bargaining model results.

Exogenously varying the outside quantity yields a list of parameters with which we calibrate the demand model. Since we compute the monopoly equilibrium after each exogenous increase of the outside quantity, some of the initial parameters in the parameter list vary along with the control variable. Specifically, the location parameters, elasticities and marginal cost. Figure 1 shows how the cross-price- and own-price elasticities relate to a changing aggregate elasticity. Exogenously increasing the quantity of the outside good to increase aggregate elasticity causes cross-price- and own-price elasticity to move in opposite directions. We observe cross-price elasticity decreasing and thus becoming less elastic, and own-price elasticity increasing becoming more elastic as aggregate elasticity increases. The increased substitutability of the inside goods with the outside good causes the inside goods to become less substitutable but also more responsive to a change in its own price.

As a result of the initial prices being fixed, the change in aggregate elasticity also causes a change in marginal cost. The increased substitutability indicates that profit margins should decrease, so that this is achieved by an increasing marginal cost. This is clearly observed in figure 2. For the 1×2 case, figure 2 shows the symmetrical marginal cost for both downstream firms. In the 2×1 setting, figure 2 shows the marginal cost for the only downstream firm. In both cases, marginal cost approaches the price as the aggregate elasticity increases. Finally, a flat logit (i.e a nest strength parameter of zero) demand function is assumed for this calibration.

Figure 1: Calibration of elasticities vs ae

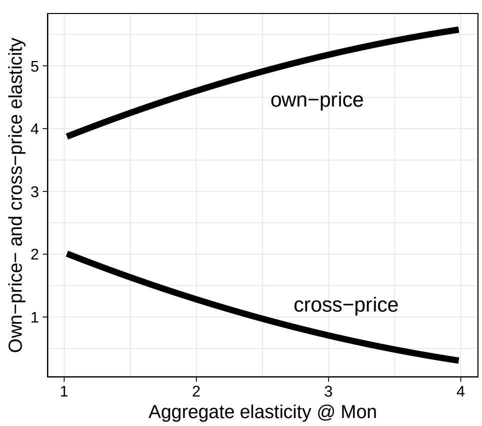
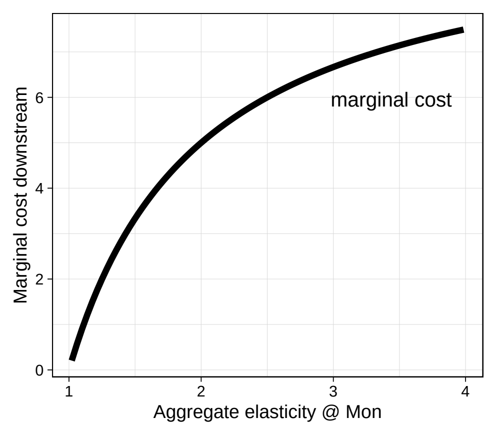


Figure 2: Calibration of marginal cost vs ae



4 Results

We discuss the results of the different models grouped according to the type of bargaining assumed. Competition, monopoly, derived demand (DD) and vertical merger(VMDD) under derived demand are grouped together since no bargaining is assumed in these models. Nash-in-Nash (NiN1), Nash-in-Shapley (NiS1) and vertical merger(VM1) under one-part pricing are grouped together since bargaining in these models is over linear wholesale prices. Finally, Nash-in-Nash(NiN2), Nash-in-Shapley(NiS2), Nash-in-Nash quantity(NiNQ) and vertical merger(VM2) under two-part pricing are grouped together since bargaining is over wholesale price and - fixed fee.

The benefits of working in a 1×2 or 2×1 bargaining setting is that we only have to consider the agreement between the vertically integrated- and rival firm post-merger. This renders the NiN and NiS specified threat points equal so that the post-merger equilibrium of these models show the same results. This enables the analysis of some key differences in NiN and NiS bargaining that we would otherwise not be able to observe.

A vertical merger (regardless of a 1×2 or 2×1 setting) leads to two opposing competitive effects. For the firm that is vertically integrating, elimination of double marginalization takes place. This procompetitive effect sees the wholesale price charged to the downstream firm pre-merger, being erased post-merger. For the post-merger rival firm, the anticompetitive raising of rival's cost occurs in a 1×2 setting. This is when the vertically integrated upstream firm increases the wholesale price to its now rival. Similarly, in the 2×1 setting, reducing of rival's revenue occurs when the vertically integrated downstream firm decreases the wholesale price it pays the upstream rival.

The pro- and anticompetitive effects on wholesale price have obvious implications for individual prices and quantities. In both a 1×2 and 2×1 setting, a vertical merger leads to an increase in the quantity and a decrease in the downstream price of the good of the vertically integrating firm. Concurrently, it leads to a decrease in quantity and a increase in the downstream price of the rival firm[2].

The simultaneous occurrence of two opposing competitive effects means that we have to evaluate the system as a whole before we can make a call on the likely merger effects. We therefore shift the focus to investigate total merger effects which are most easily observed in the figures for total quantity (figures 3 and 4)[3]. In the subsections that follow, we discuss these effects by the assumed bargaining and thus, focus on one panel at a time.

[2]All of these effects can be observed in the appendix in figures 7(a), 10(a), 7(b), 10(b), 6(c), 6(d), 9(c), 9(d), 6(a), 6(b), 9(a) and 9(b)

[3]The change in total quantity closely tracks the change in consumer surplus so that welfare effects are inferred from either of these measures.

Figure 3: Total quantity vs ae 1 × 2 setting

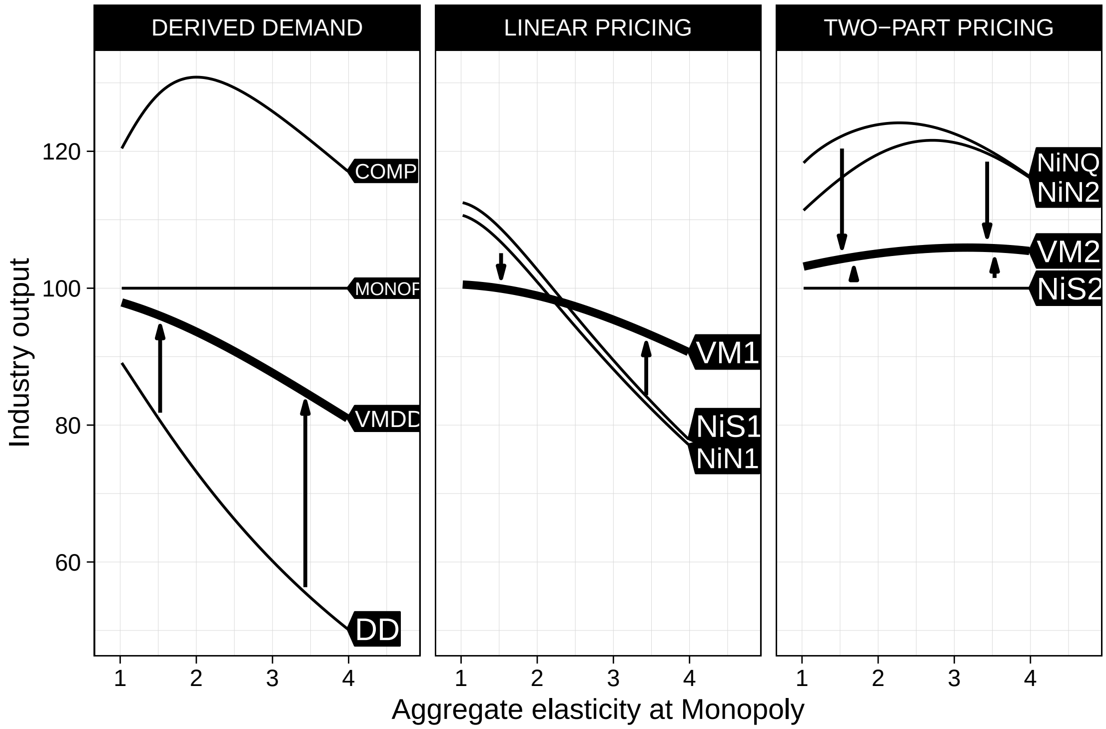
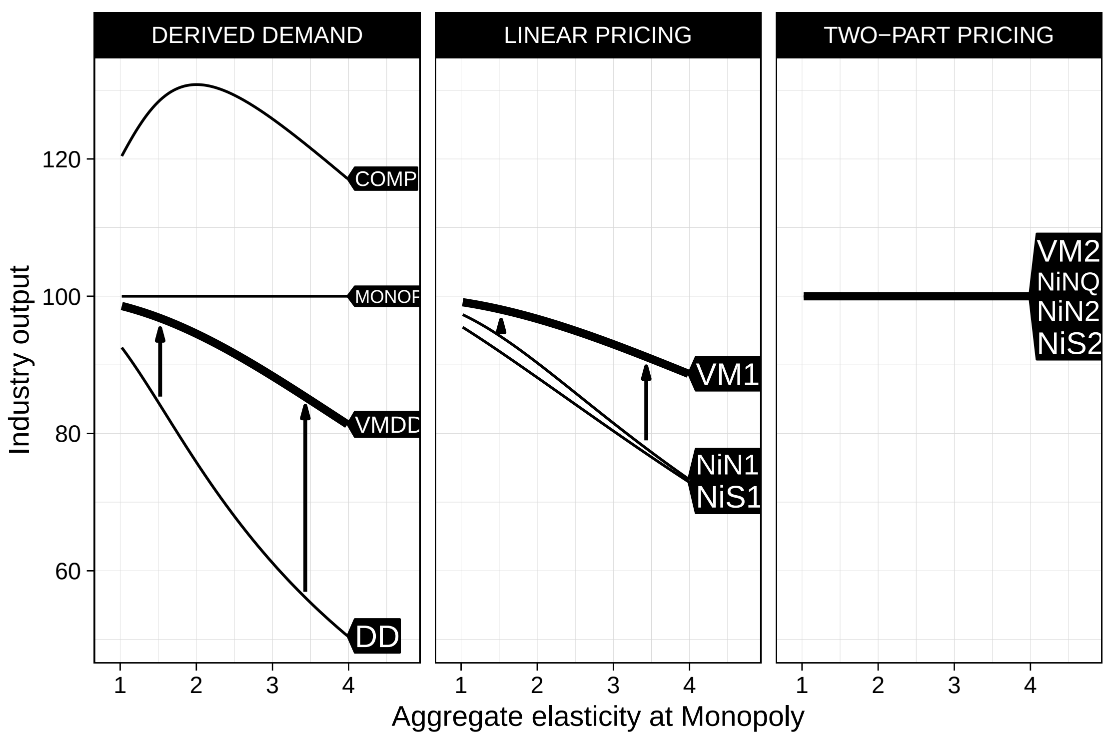


Figure 4: Total quantity vs ae 2 × 1 setting



4.1 What do we know: Derived demand merger effects

In the first panel of figures 3 and 4 we show the traditional derived demand model for a vertical relationship. We also show the competition and monopoly cases as benchmark models. Along the DD locus, the upstream firm makes a take-it-or-leave-it linear wholesale price offer to the downstream firm. This contributes to the downstream firm's total marginal cost who then marks it up again. As a result, output is way below the monopoly output.

Along the VMDD locus, we have the post vertical merger world. The upstream firm still makes a take-it-or-leave-it offer to the unintegrated firm, but the downstream equilibrium is no longer symmetric. The captive downstream manufacturer gains share because the perceived margin on sales to its product is bigger than the margin on sales to its rival. As a result, it can decrease the price of its own product, which increases its output but reduces sales of rivals. The substantial increase in the vertically integrated quantity outweighs the decrease in rival's quantity leading to an increase in total output.

The up arrows from the DD to VMDD loci show us what we already know: A vertical merger in a derived demand setting moves the market closer to monopoly by raising total quantity which closely tracks the change in consumer surplus. The derived demand model is then only able to find a procompetitive merger as the elimination of double marginalization always outweigh the anticompetitive effect (raising of rival's cost in 1×2 setting and reducing rival's revenue in a 2×1 setting). This is completely in line with the findings of previous studies as mentioned in the introduction.

Table 1: Derived demand summary: Figures 3 and 4 panel 1

Model: Comment	**Rules**	1×2**:** Results	2×1**:** Results
DD: Derived Demand models generalize the old "successive monopoly" models, by allowing a more general downstream game, e.g. Bertrand or Cournot.	Upstream firm(s) set wholesale linear prices and downstream firm(s) play a noncooperative game taking wholesale prices as given.	Pre-merger output way below monopoly output due to big double marginalization.	Pre-merger output way below monopoly output due to big double marginalization.
VMDD: The merged firm eliminates double marginalization (EDM) and raises rival's cost (RRC) or reduces rival's revenue (RRR) to unintegrated retailer.	Vertically integrated firm sets a wholesale linear price to unintegrated firm, then plays noncooperative game with same in downstream (or upstream) market.	Post-merger output slightly below monopoly output. Vertical merger raises output because EDM > RRC.	Post-merger output slightly below monopoly output. Vertical merger raises output because EDM > RRR.

4.2 Linear pricing: One instrument, two conflicting goals

In the second panel of figures 3 and 4 we introduce bargaining over linear prices. Along the NiN1 and NiS1 loci we assume bargaining over linear wholesale prices of goods. Downstream firms subsequently set retail prices in Nash equilibrium. Each bilateral bargain is assumed to be reached on the basis of a Nash bargaining solution relative to either a NiN1 or NiS1 specified threat point. For the NiN1 model, the threat point for each agreement (the outcome should an agreement not be reached) is taken to be the continuation of the other agreement at the same wholesale price. The NiS1 model allows for bilateral contracts that are contingent on which other agreements are made (renegotiation) and thus the threat point specifies for an agreement at a new wholesale price that satisfies the Nash bargaining solution for a single good.

Despite differences in threat points, these two models are almost indistinguishable for all of the variables of interest. This illustrates the theme of this section: In linear pricing, there is only one instrument - the wholesale price - to achieve two conflicting goals. Seeing that in this setting the wholesale price makes out a significant part of the marginal cost of the downstream firms, it helps determine the downstream prices, quantities and ultimately profits. In turn, the downstream equilibrium determines the wholesale quantities that the upstream firm will sell. The wholesale price is thus firstly an instrument that the upstream firm can use to increase the industry profits. However, concurrently the wholesale price is also the only instrument with which the upstream firm takes its share of the profit. These two goals work in opposite directions seeing that a lower wholesale price is better for the first goal, but a higher wholesale price fits the second goal. These counteracting effect then diminish the difference in threat points between NiN1 and NiS1 so that we do not observe any discernible difference between these two models.

Apart from the conflicting goals of the upstream firm, the introduction of bargaining means that the downstream firms also influence the wholesale prices. Consequently, firms are able to bargain for a lower wholesale price than the take-it-or-leave-it scenario in a derived demand setting[4]. As a result, we observe total quantity being higher than in a derived demand setting for both industry structures (figures 3 and 4).

The VM1 locus shows the post vertical merger world for both the NiN1 and NiS1 models. The merged firm negotiates a wholesale price with the now-rival firm in a Nash bargaining setting. Subsequently, asymmetric retail prices are determined in a Nash equilibrium. Again, the procompetitive elimination of double marginalization increases output for the vertically integrated firm. The anticompetitive raising of rival's cost (1×2 setting) or reducing of rival's revenue (2×1 setting) reduces output for the rival. The relative magnitude of these effects determine the ultimate predicted effects of a vertical merger.

The linear pricing 1×2 setting is the only case in our simulations where the competitiveness of a merger is dependent on the level of aggregate elasticity. It is in cases like this where the benefit of doing numerical simulations is highlighted as we can ascertain the relative magnitude of merger effects. We find that following a vertical merger in a low aggregate elasticity setting, total quantity (figure 3) in the market decreases. At low aggregate elasticity, pre-merger wholesale prices are high. Despite the fact that this results in a significant elimination of double marginalization post-merger, the vertically integrated firm also manages to significantly raise its rival's cost. This latter effect outweighs the procompetitive effect and thus yield a welfare-decreasing (anticompetitive) vertical merger. At high

[4]This can be observed when comparing panel 1 and 2 of figures 7(a) and 7(b) and figures 7(a) and 7(b).

aggregate elasticity, pre-merger wholesale prices are significantly lower, so that the elimination of double marginalization post-merger is smaller. However, it still outweighs the smaller increase in the rival's wholesale price so that the model predicts a welfare-increasing (procompetitive) vertical merger.

The linear pricing 2×1 setting shows a procompetitive merger for all levels of aggregate elasticity. However, it does relate to the 1×2 case in that a merger is more procompetitive at a higher aggregate elasticity. Compared to the 2×1 derived demand merger, a merger is less procompetitive for every level of aggregate elasticity.

Table 2: Linear pricing summary: Figures 3 and 4 panel 2

Model: Comment	**Rules**	1×2**:** Results	2×1**:** Results
NiN1: One instrument (linear wholesale price) performs two tasks: determines the size of total profit and how profit is split. NiN1 and NiN2 are very close.	Upstream and downstream players bargain bilaterally over linear wholesale price, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements.	Output above monopoly output for low aggregate elasticity and strong nest strength.	Output always below monopoly output in range of parameters we consider. Not true in general.
NiS1: As above, only one instrument (linear wholesale price) performs two tasks, but the alternatives to agreement change.	Upstream and downstream players bargain bilaterally over linear wholesale price, but expect prices to change if agreements don't make. Threat point determined by re-negotiating remaining agreements.	Output above monopoly output for low aggregate elasticity and strong nest strength.	Output always below monopoly output in range of parameters we consider. Not true in general.
VM1: The merged firm eliminates double marginalization.	Vertically integrated firm bargains over linear wholesale prices to unintegrated firm.	In case with low aggregate elasticity or with strong nest strength vertical mergers have anticompetitive effects (RRC>EDM). The first factor results in fewer lost sales to "no purchase" alternative; the second makes it easier to capture lost sales from the unintegrated retailer. Integrated firm profit increases due to better post-merger threat point.	Output slightly below monopoly output in range of parameters we consider and vertical mergers always have beneficial effects, as EDM>RRR. Neither are true in general. Integrated firm profit increases due to better post-merger threat point.

4.3 Two-part pricing: Threat points matter

In the third panel of all the figures we introduce two-part pricing bargaining. In this setting, we assume bargaining over wholesale prices and fees. In both industry structures and specifications of bargaining types, parties wish to increase industry profits when negotiating over the wholesale price so that incentives of bargaining players are aligned. Consider the 1×2 setting: downstream firms earn profit

$$\pi_i = (p_i - mc_i - w_i) * q_i - f_i, i = 1, 2$$

and pay upstream firm(s) $w_i * q_i + f_i$ where w_i is the marginal wholesale price and f_i is a fixed fee used to ensure an equal profit split over the threat points. The upstream firm earns profit

$$\pi_A = ((w_1 - mc_{A1}) * q_1 + f_1) + ((w_2 - mc_{A2}) * q_2 + f_2)$$

The mc_{A1} and mc_{A2} are the possibly different upstream marginal costs of supplying goods to M_1 and M_2 respectively. We assume that $mc_{A1} = mc_{A2} = 0$ for convenience. In a 1×2 two-part pricing setting, A and 1 negotiate agreement $A1 = (w_1, f_1)$ while taking into consideration agreement $A2$ and A and 2 negotiate agreement $A2 = (w_2, f_2)$ taking into consideration agreement $A1$. In these negotiations, the assumed status of the other agreement characterizes the type of bargaining.

In the 2×1 setting, we invert the bargaining set up by assuming firm 1 (downstream monopolist) contracts with A and B (upstream firms) to produce goods for sale by firm 1. As above, we look at the bargaining between 1 and A who negotiate over contract $A1 = (w_1, f_1)$ while taking into consideration agreement $B1$ and 1 and B negotiate agreement $B1 = (w_2, f_2)$ taking into consideration agreement $A1$. Again, the assumptions we make about the other agreement characterize either NiN or NiS bargaining.

In a NiN setting, each surplus is maximized independently assuming that the terms of the other agreement are fixed. In contrast, NiS assumes that the total surplus from both agreements is maximized and that the effects of the other agreement are accounted for in computing the surplus from an agreement. From this, it is clear that there are now two different parameters to achieve two conflicting goals as set out in section 4.2. Thus, there is an incentive of the pivotal player (the firm involved in both bilateral bargains) to internalize competition in the opposing market (the upstream market in a 2×1 and the downstream market in a 1×2 setting) when determining the wholesale price. This incentive leads to different outcomes depending on the assumed type of bargaining.

4.3.1 ***Nash-in-Nash:*** Bargaining against yourself

In the 1x2 *Nash-in-Nash* two-part pricing setting, 1 and A negotiate agreement $A1 = (w_1, f_1)$ by assuming that agreement $A2 = (w_2, f_2)$ is fixed. To compute equilibrium, we have to check the conditions under which A and 1 can increase joint profit by reaching a different agreement. This occurs only if a change leads to an increase in their joint profit.

$$\Delta(\pi_A + \pi_1) = \Delta(q_1 * (p_1 - mc_1) + w_2 * q_2) > 0$$

Note that the wholesale payments cancel each out, as they are revenue to A but costs to 1.

Intuitively, A and 1 try to make themselves better off at the expense of 2. Of course, when A and 2 negotiate, they try to do the same thing. NiN equilibrium occurs at a point

Table 3: Two-part pricing summary: Figures 3 and 4 panel 3

Model: Comment	**Rules**	1×2**:** Results	2×1**:** Results
NiN2: The NiN assumption that the other agreement is fixed make the parties bargain as if they don't know that the bargain they strike in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over two part prices, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements.	Output above monopoly output as NiN assumption makes it appear that the single upstream firm is bargaining against itself. Output is above monopoly output.	Pre-merger output equals monopoly output because downstream monopoly retailer internalizes upstream competition.
NiNQ: The NiN assumption that the other agreement is fixed make the parties bargain as if they don't now that the bargain they strike in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over fixed wholesale price and quantity, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements.	Output above monopoly output as NiN assumption makes it appear that the single upstream firm is bargaining against itself. Output is above monopoly output.	Pre-merger output equals monopoly output because downstream monopoly retailer internalizes upstream competition.
NiS2: Parties bargain as if they know that they will get a share of any improvement in profit in grand coalition (both agreements make). They are willing to, e.g., reduce wholesale price if that leads to higher total profit.	Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements don't make. Threat point determined by renegotiating remaining agreements.	Output equals monopoly profit.	Pre-merger output equals monopoly output because downstream monopoly retailer internalizes upstream competition.
VM2: The merged firm eliminates double marginalization, favours its captive downstream retailer in 1×2 case, but not 1x2 cases.	Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements don't make. Threat points determined recursively by profits in set of agreements without current agreement.	Output above monopoly profit because of what Church (2008) calls "inefficient contracting," i.e., the increased margin on the integrated product due to EDM gives the integrated firm an incentive to increase its sales. NiN2: Vertical mergers have big negative effect. NiNQ: Vertical mergers have big negative effect. NiS2: Vertical mergers small positive effect.	Post-merger output equals monopoly output because downstream monopoly retailer internalizes upstream competition. Vertical Mergers have no effect.

when it is no longer profitable for either of the pairs $(A,1)$ or $(A,2)$ to deviate from the agreement,

$$w_1^* = \underset{mc_1}{\text{argmax}}(\pi_A + \pi_1)$$

The fixed fee f_1 is chosen to split surplus by maximizing the product of the surpluses,

$$f_1^* = \underset{f_1}{\text{argmax}}(\pi_A - \pi_A^*)(\pi_1 - \pi_1^*).$$

In the case of transferable utility this reduces to

$$\pi_A - \pi_A^* = \pi_1 - \pi_1^*,$$

which is exactly why both A and 1 want to maximize $\pi_A + \pi_1$ at the previous step. Here $\pi_1^* = 0$ but $\pi_A^* = (w_2^* - mc_2) * q_2^* + f_2$ with only q_2^* manufactured by 2.

What differentiates NiN from NiS is the suboptimal form of the NiN contracts, as they would remain fixed even if the other agreement does not make.

In the NiN equilibrium $\{A1^*, A2^*\}$, the suboptimal nature of the contractual form shows up in joint output above $(q_1^* + q_2^*)$ and joint profit $(\pi_1^* + \pi_2^*)$ below monopoly levels. NiN leads the pair $(A,1)$ to compete with $(A,2)$, lowering wholesale prices to maximize (almost) independent profits in Nash equilibrium, with the dubious consequence that A in the two negotiations ends up competing with itself.

In the 2×1 NiN setting, we now look at the bargaining between 1 and A who negotiate by assuming that agreement B1=(w_2, f_2) is fixed, in which 1 pays $w_2 * q_2 + f_2$ to B, depending on the quantity q_2 that 1 chooses to sell to maximize its final retail profit. In contrast to the 1×2 setting, the equilibrium is the joint profit maximizing outcome.

To see this, we have to show that neither firm has an incentive to change wholesale prices from marginal costs, $w_i = mc_i, i = A, B$. If w_1 were lower than mc_A, 1's total operating profit would increase but the total joint profit $\pi_1 + \pi_A = (p_1 - mc_A) * q_1 + (p_2 - mc_B) * q_2$ would decrease as 1 sets retail prices to maximize $\pi_1 = (p_1 - w_1) * q_1 + (p_2 - mc_B) * q_2$. This would result in a price lower than the monopoly price for p_1. And similarly, if 1 and A were to raise w_1 above mc_A.

We see that the pair $(1, A)$ has no incentive to deviate from the $w_1 = mc_A$ marginal price, and likewise $(1, B)$ will not deviate from $w_2 = mc_B$. The downstream firm 1 takes these wholesale prices as given and finds the monopoly retail prices and quantities maximizing joint surplus ($= \pi_1 + f_1 + f_2$) given that f_1 and f_2 are fixed.

From the above, it is clear that NiN bargaining leads to results that depend on the industry structure as a result of the wholesale prices being cost-based. This is in line with the literature reviewed in the introduction. While the pivotal player manages to completely internalize competition in the opposing market in the 2×1 setting, it is unable to do so in the 1×2 setting. In the latter, we see the NiN models showing a more competitive outcome than the NiS2 model. Moreover, fixing the quantity and total price instead of the wholesale price and fee, as in the NiNQ model, leads to an outcome even closer to competition. The difference between NiNQ and NiN2 here is that the joint profit functions for NiN2 for $(A, 1)$ includes q_2, and $(A, 2)$ includes a term of q_1. These terms "internalizes" some of the "schizophrenia" (Collard-Wexler et al., 2019) associated with A bargaining against itself, as the pairs will not compete as vigorously against each other as in NiNQ, where there is no such dependence. In NiNQ the joint profit of $(A, 1)$ is not a function of q_2, and $(A, 2)$ is not a function of q_1. This leads to the more intense competition between the pairs.

4.3.2 *Nash-in-Shapley:* Internalizing competition

When bargaining in NiS, the pivotal player takes full cognizance of the externality that the other agreement imposes on this bargain. For example, in the 1×2 setting, we assert that firms A and 2 would anticipate the change in conditions if agreement $A1$ fails and set a (w_2, f_2) for this contingency, different from the contract when agreement $A1$ makes. This changes the threat point in negotiation with 1. Moreover, we assert that firms A and 2 would anticipate how the split of profits determined by f_2 would change (through renegotiation) as they vary w_1. This leads to higher wholesale prices signaling the downstream firms to price at the joint profit maximizing level. Therefore, NiS achieves the joint profit maximizing outcome.

In the 2×1 NiS case, the pair $(1, A)$ anticipates the split in profits 1 will realize with B, and so will set w_1 to maximize the total surplus $\pi_1 + \pi_A + \pi_B = q_1 * (p_1 - mc_A) + q_2 * (p_2 - mc_B)$. And $(1, B)$ will maximize the same total surplus. Since 1 will set retail prices to maximize $q_1 * (p_1 - w_1) + q_2 * (p_2 - w_2)$, both A and B are happy to set $w_1 = mc_A$ and $w_2 = mc_B$, leading to monopoly retail prices, and collect their share of the maximum possible total surplus.

It is not the result of the industry structure, where operating profits are earned or the marginal cost balance between upstream- and downstream firms that yields the NiS2 model equal to the monopoly outcome. Rather, it is a direct result of how the model characterizes bargaining. NiS2 assumes that total surplus from both agreements is maximized and that the effects of the other agreement are accounted for in the calculation of the surplus. It is then exactly the monopoly outcome that maximizes total surplus when determining the wholesale price. Hence, we observe the NiS2 model follow the monopoly outcome in both industry settings (figure 3 and 4). This is robust against a change in the pivotal player, where the operating profit is earned and what the marginal cost balance between the pivotal- and other players is. The NiS model thus displays an independence of different industry specifications and provides a solution to the opportunism problem as reviewed in the introduction.

4.3.3 Merger effects and drivers in two-part pricing

The VM2 locus in the third panel for figures 3 and 4 shows the NiN2, NiNQ and NiS2 models for a vertical merger. Along this locus, the merged firm negotiates a wholesale price and fee with the now-rival firm in a Nash bargaining setting. Subsequently, retail prices (now asymmetric) are determined in a Nash equilibrium. Again, the procompetitive elimination of double marginalization increases output for the vertically integrated firm. The anticompetitive raising of rival's cost in a 1×2 setting and reducing rival's revenue in a 2×1 setting, reduces output for the rival. The relative magnitude of these effects determine the ultimate predicted competitiveness of a vertical merger.

Moving away from monopoly

A vertical merger usually means a move towards monopoly. However, the NiS2 model in both industry settings and NiN models in a 2×1 setting is already at monopoly pre-merger. Following a merger, it is only the NiS2 model in a 1×2 setting that moves away from monopoly towards a more competitive outcome.

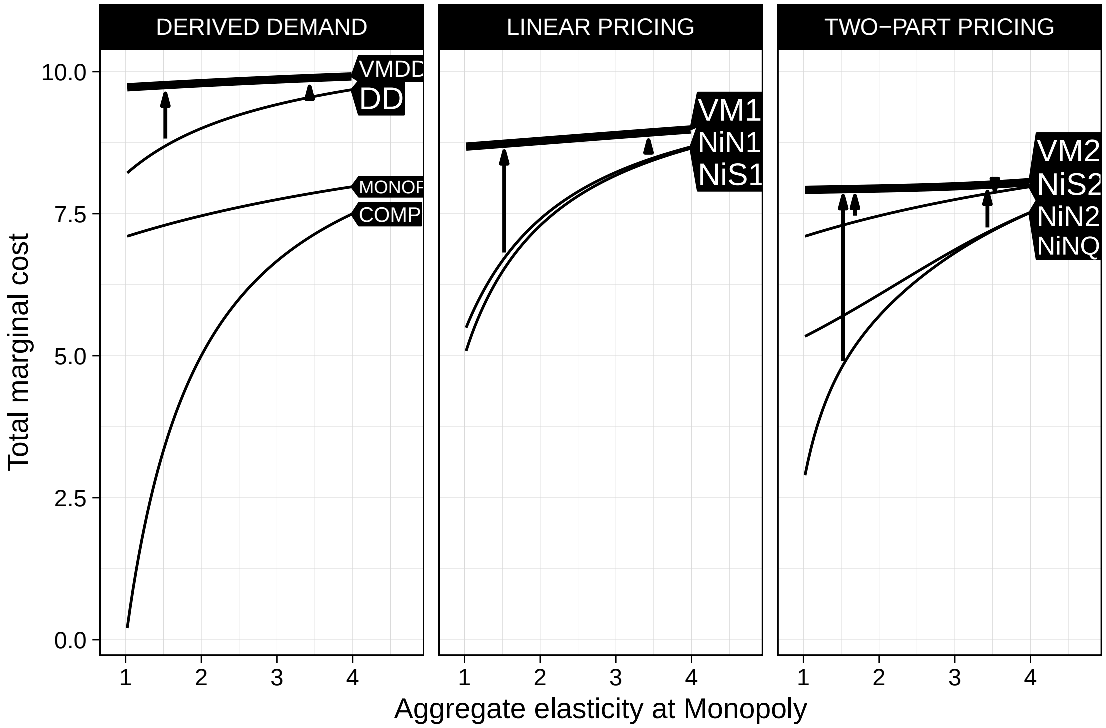
Post-merger in a NiS2 1×2 model, the double margin of the vertically integrated firm is eliminated but the pivotal player (formerly the upstream firm) also inherits the marginal cost from its vertically integrated downstream firm[5]. It now acts as if this, and not the transfer price (which is zero by assumption), is its true marginal cost. In contrast, the rival downstream firm's total marginal cost is partly determined by negotiation with the vertically integrated firm. In this negotiation, the vertically integrated firm cannot effect a commitment to raise retail prices to monopoly prices, as we assume firms are prohibited from setting retail prices as part of their negotiations. Moreover, the vertically integrated firm cannot credibly commit to the price at the monopoly level because of the change in its marginal cost. Consequently, the vertically integrated firm reduces its price and increases quantity (figures 6(c) and 6(a)).

A vertical merger in a two-part pricing setting shows the lowest post-merger wholesale prices of all the models allowing a vertical merger in the 1×2 case (figure 5). Seeing that the NiS2 pre-merger wholesale price is higher than the NiN counterparts, the raising rival's cost effect for NiS is diminished. As such, we observe some of the smallest increases in wholesale price in figure 7(b). Hence, the decrease in post-merger quantity for downstream firm 2 (figure 6(b)) is also not as significant as in some other models. The combination of this with a greater increase in quantity of the vertically integrated firm (figure 6(a)) sees the total quantity increasing following a vertical merger in a NiS2 setting (figure 3).

Because the pivotal player is downstream and it does not inherit a marginal cost post-merger, we do not see a merger effect on total quantity in the 2×1 setting. All three models remain at the monopoly equilibrium so that there is no procompetitive elimination of double marginalization or anticompetitive reducing of rival's revenue in this setting.

[5] Recall that we fixed the pre-merger marginal cost of the upstream firm to zero and it was able to induce monopoly prices and quantities by setting the wholesale prices to both downstream firms.

Figure 5: Total marginal cost of firm 2 vs ae 1 × 2 setting



Moving towards monopoly

The NiN two-part pricing models in a 1×2 setting are the only models in two-part pricing that show an anticompetitive merger. This is attributed to the pivotal player in the NiN models bargaining against them self as explained in section 4.3.2. As a result of this, we observe the lowest total marginal costs (figure 5) of all the models.

Following a vertical merger, the incentive of the upstream firm to internalize competition between its vertically integrated downstream firm and the rival firm is eliminated. However, it cannot reach the monopoly outcome as in the 2×1 case for two reasons. Firstly because the pivotal player is now upstream and thus it cannot impose monopoly retail prices as a result of the order of profit maximization[6]. Secondly, the vertically integrated firm has an incentive to raise its rival's cost so that we observe a substantial percentage increase in the post-merger wholesale price.

The 1×2 two-part pricing setting is unique in that the specification of the type of bargaining (NiN or NiS) completely predetermines the competitiveness of a vertical merger.

[6] In the 2×1 case, the pivotal player is downstream and retail prices are determined after wholesale prices. It is thus possible to end up at the monopoly outcome post-merger as well.

5 Conclusion

We have come along way since the old successive monopoly models of vertical merger. Replacing the old take-it-or-leave-it bilateral interactions with more realistic models of bargaining adds a considerable degree of complexity and nuance. Exercises like the ones in this paper can help unpack this complexity and identify the drivers of competitive effects. For determining competitive effects, we are able to identify the assumptions that matter and why they matter from the existing literature. However, this paper adds to our understanding of how much they matter.

The results may prove useful as a guide to empirical or theoretical work that can help us identify which bargaining model is most appropriate in a given setting. They may also be useful to enforcement agencies and those appearing before them as they can identifying assumptions and conditions that lead to anticompetitive effects.

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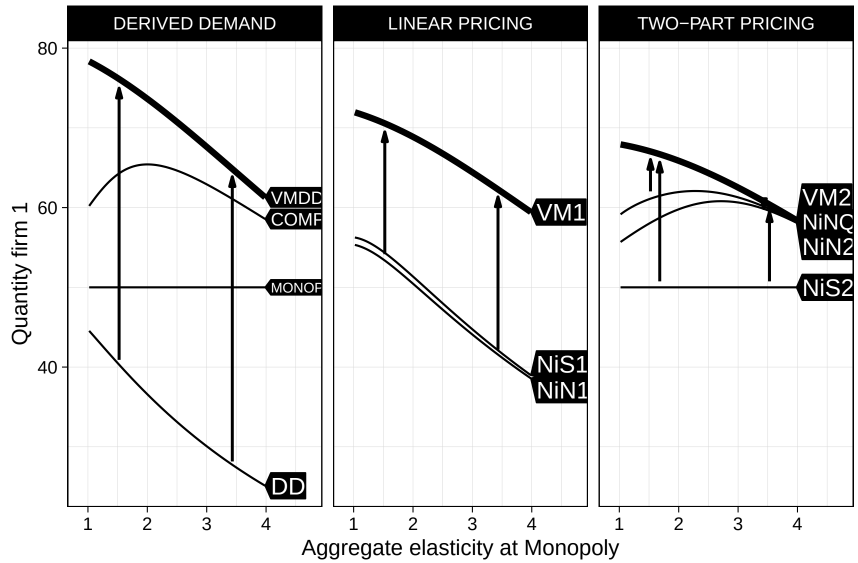
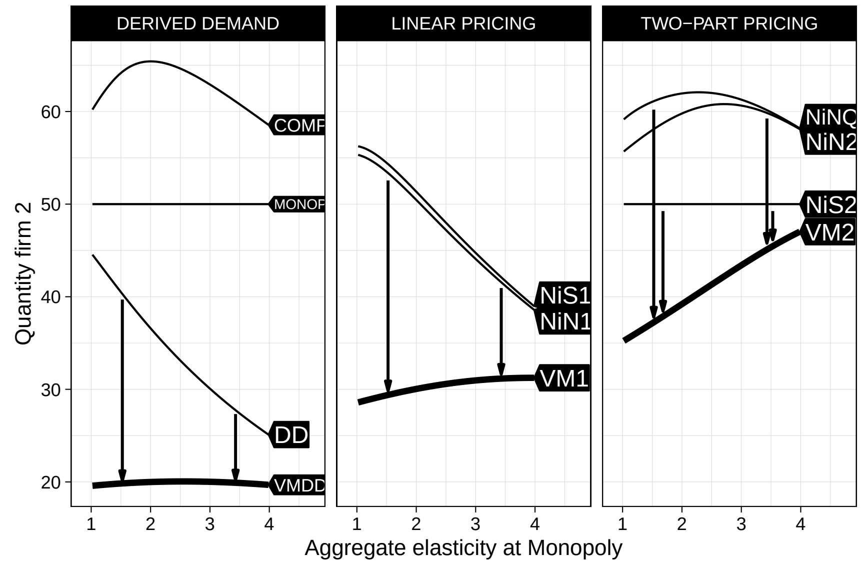


(a) Downstream quantity firm 1 vs ae



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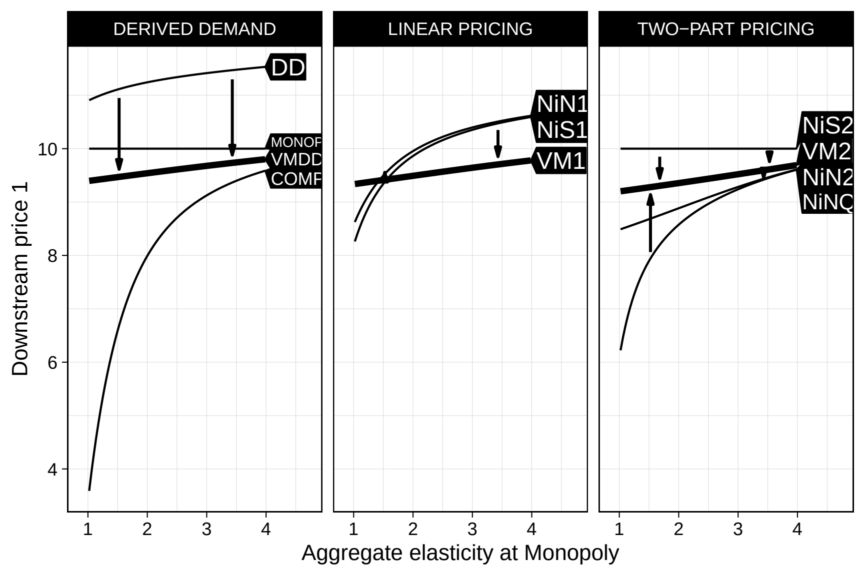

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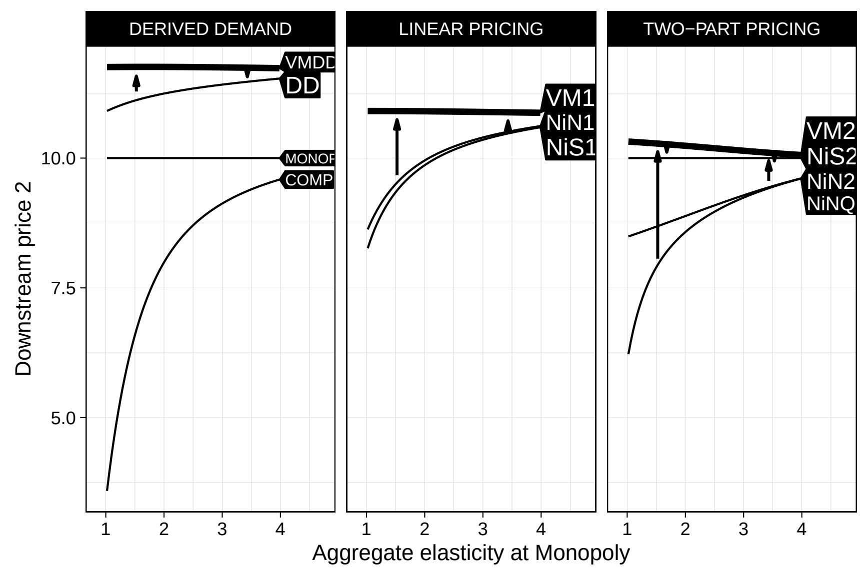
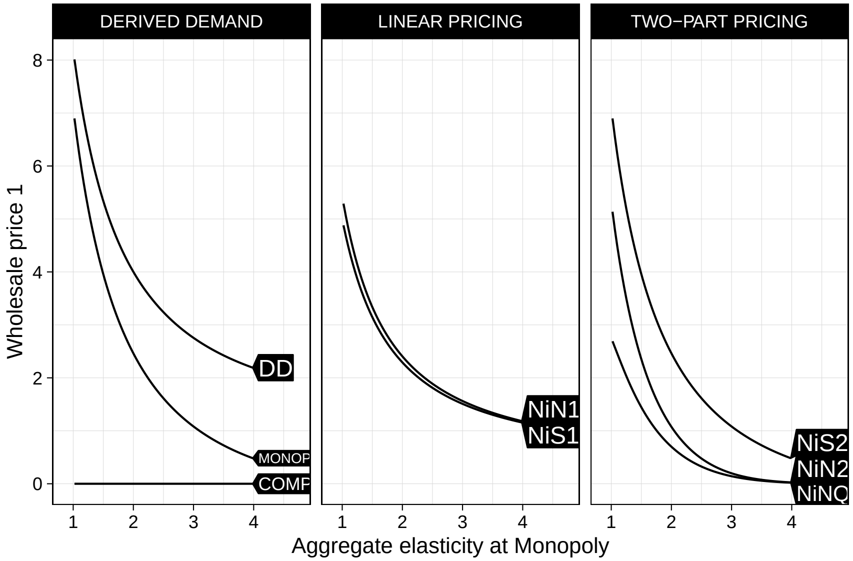
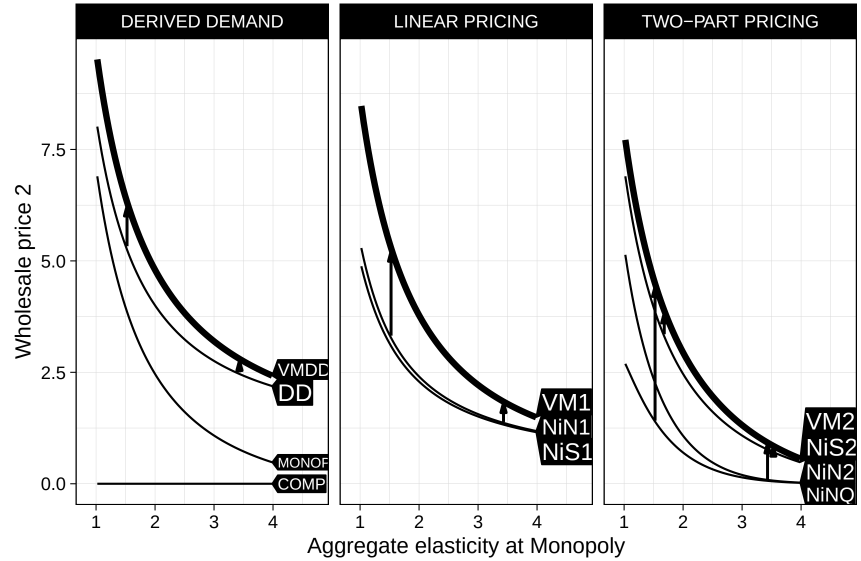


(d) Downstream price firm 2 vs ae

Figure 6: Retail prices and quantities for a 1x2 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



(a) Wholesale price firm 1 vs ae



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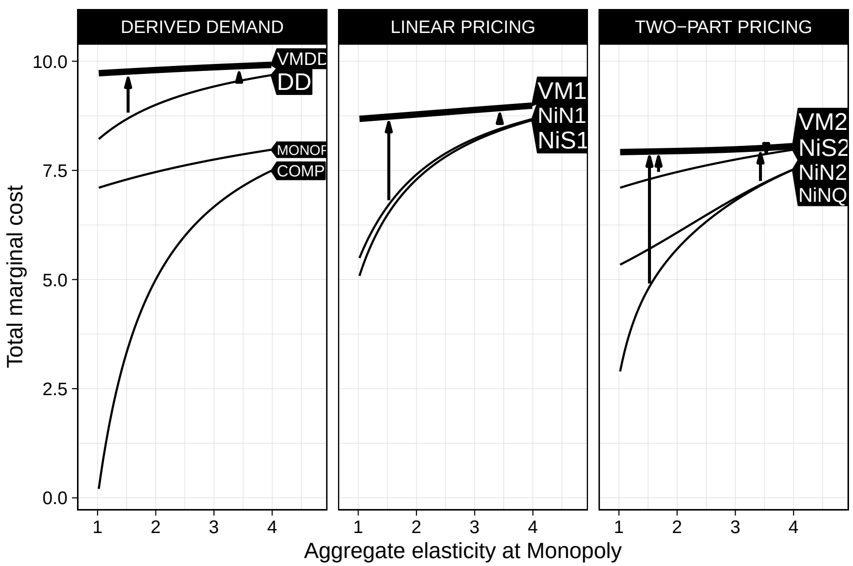
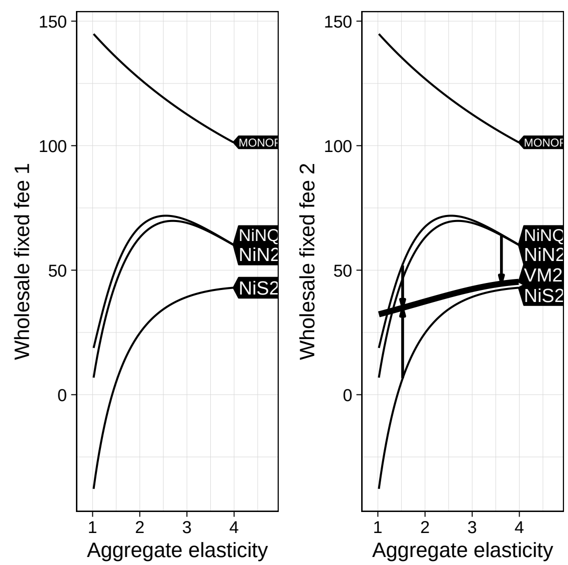
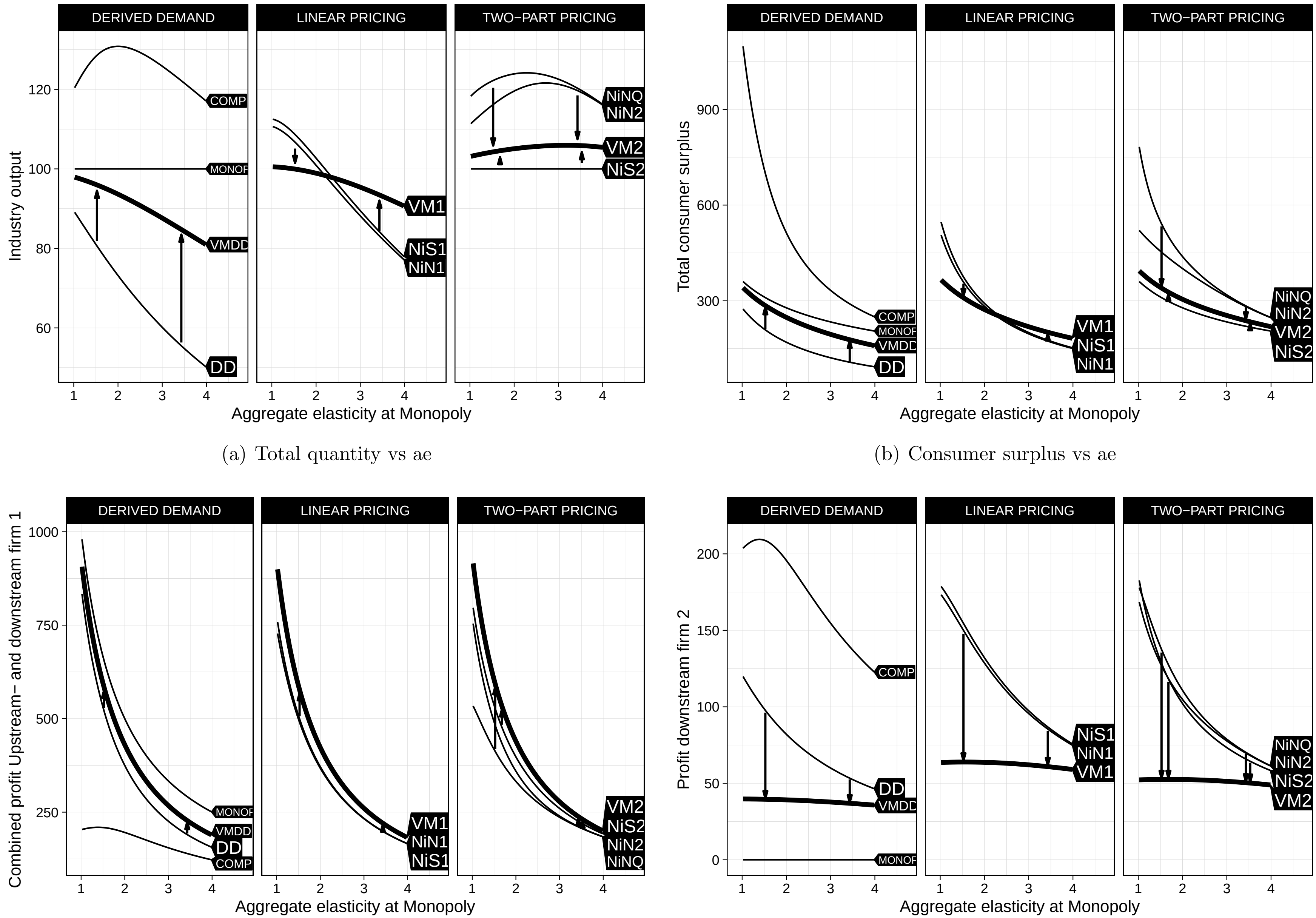


(c) Total marginal cost downstream firm 2 vs ae



(d) Wholesale fees for firm 1 and 2

Figure 7: Wholesale prices and fees for a 1x2 setting: (a) shows the elimination of double marginalization for the vertically integrated firm; (b) shows the raising of rival's cost for the rival firm; (c) shows the increase in total marginal cost for the rival firm; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting



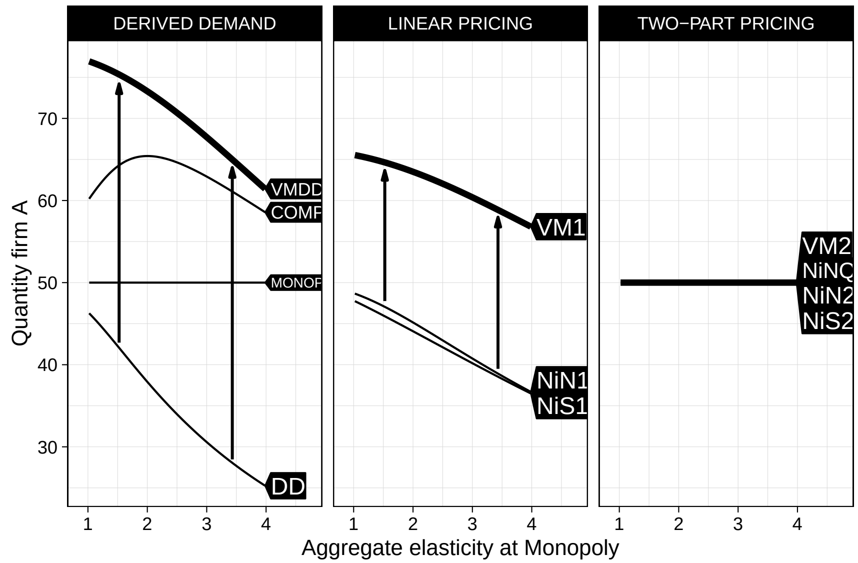
(a) Total quantity vs ae

(b) Consumer surplus vs ae

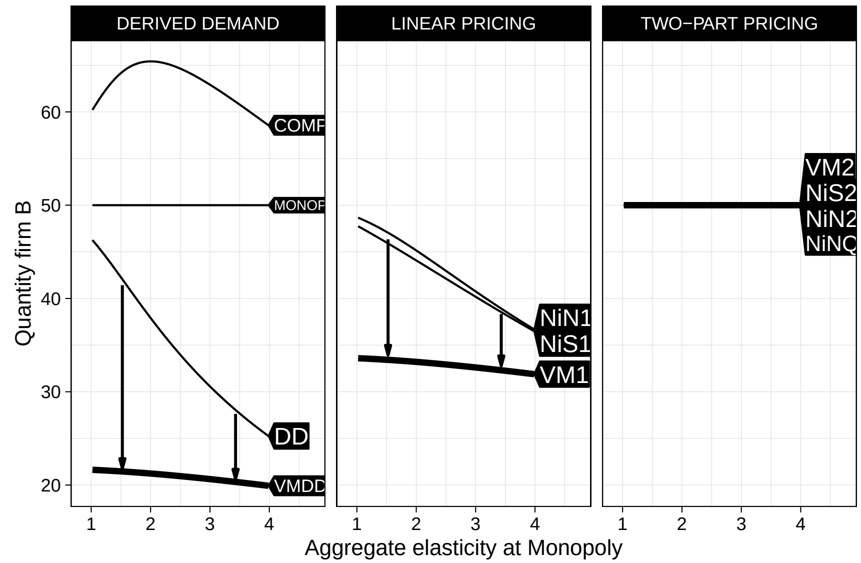
(c) Combined profits firm A and 1 vs ae

(d) Profit downstream firm 2 vs ae

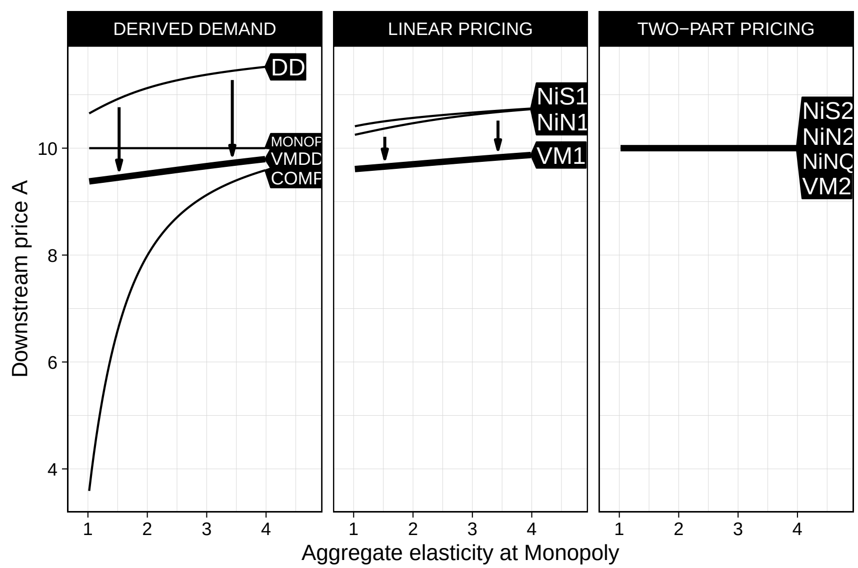
Figure 8: Profit and welfare in a 1x2 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the combined profits of the upstream firm and downstream firm 1 - post-merger profit is always higher than pre-merger combined profit; and, (d) shows the profit of firm 2 - post-merger profit is always lower than pre-merger profit



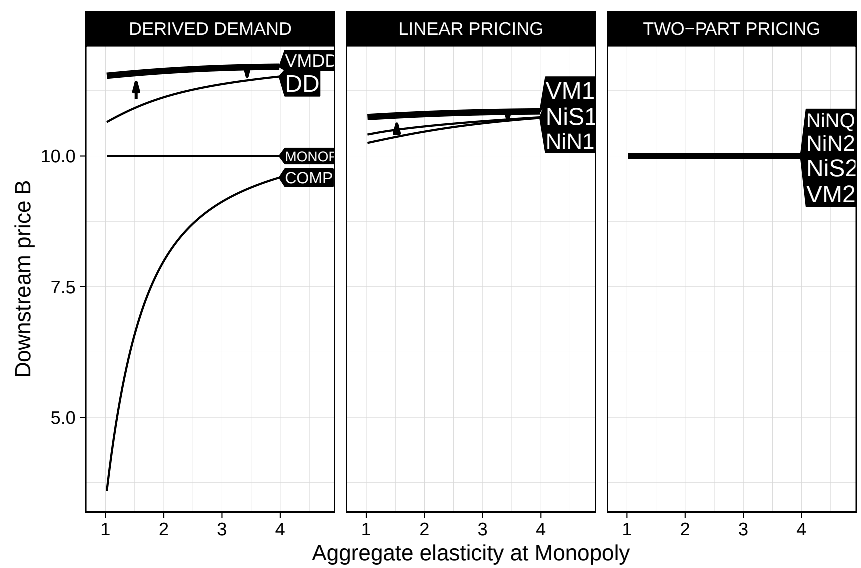
(a) Downstream quantity firm 1 vs ae



(b) Downstream quantity firm 2 vs ae

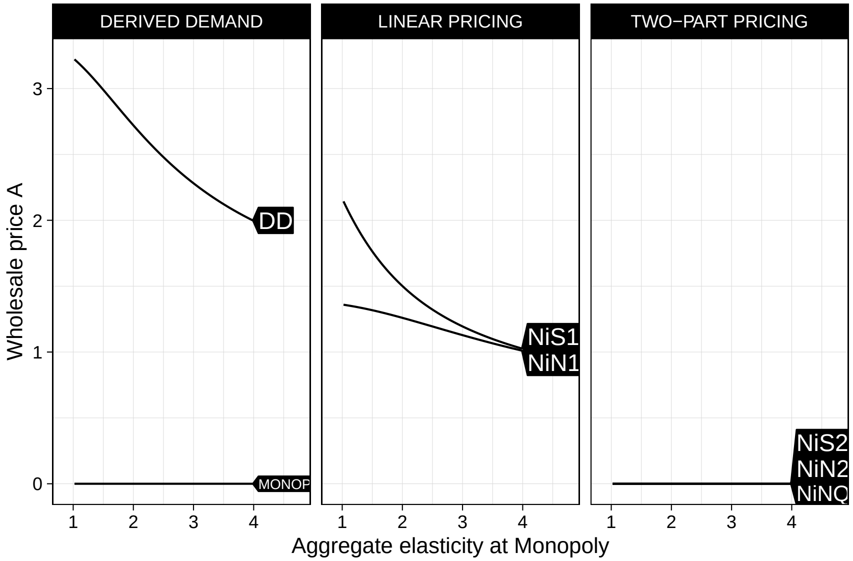


(c) Downstream price firm 1 vs ae

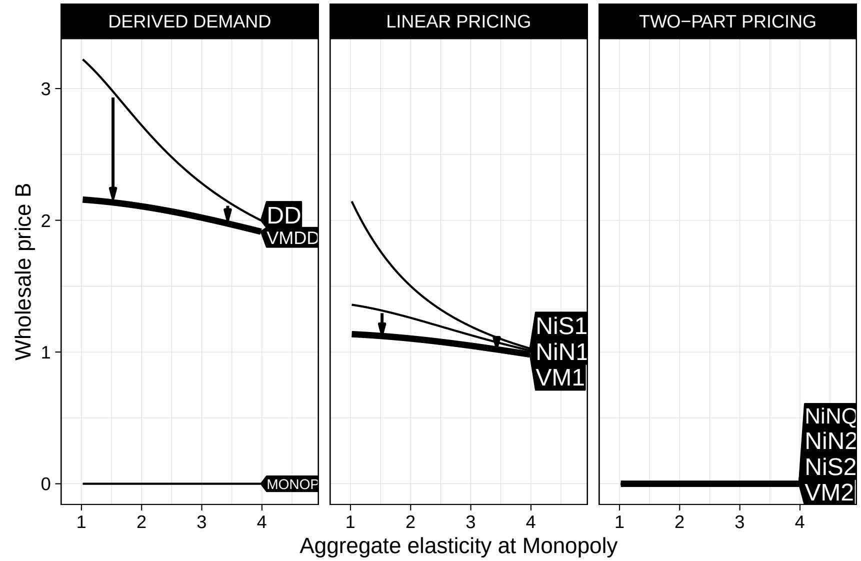


(d) Downstream price firm 2 vs ae

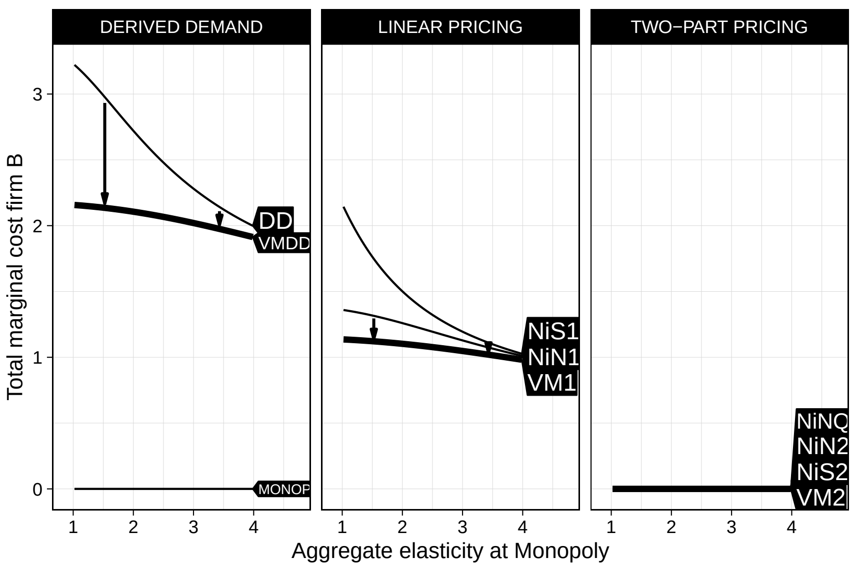
Figure 9: Retail prices and quantities for a 2x1 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



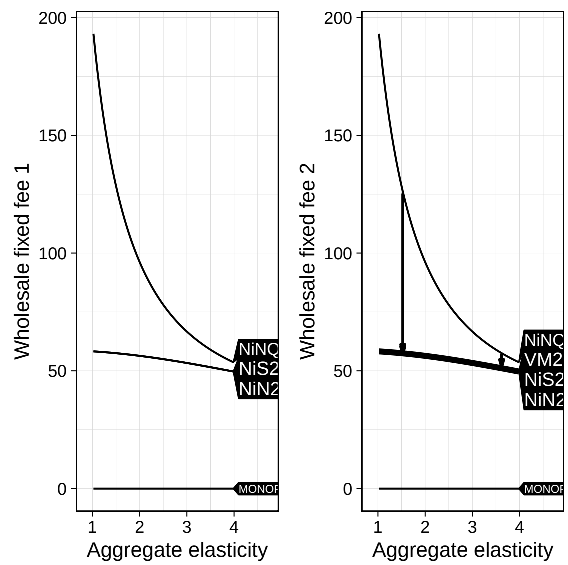
(a) Wholesale price firm 1 vs ae



(b) Wholesale price firm 2 vs ae

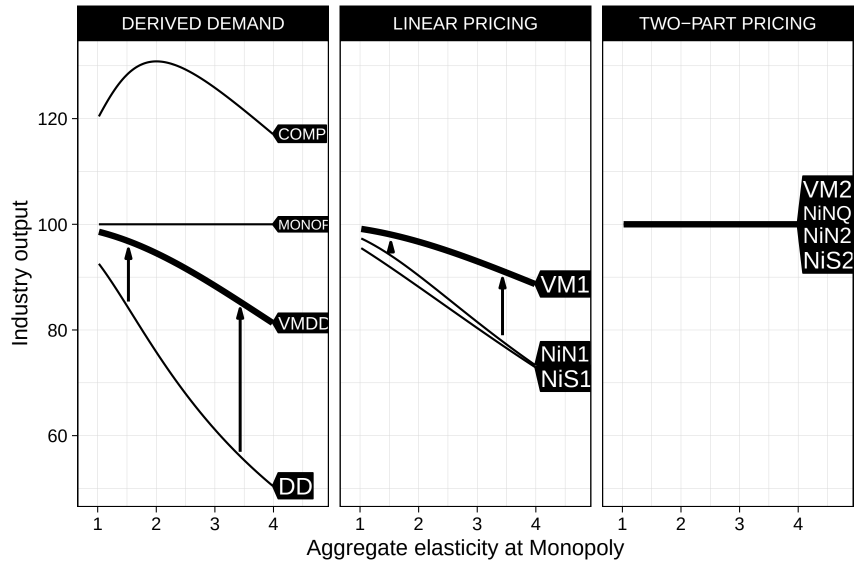


(c) Total marginal cost downstream firm 2 vs ae

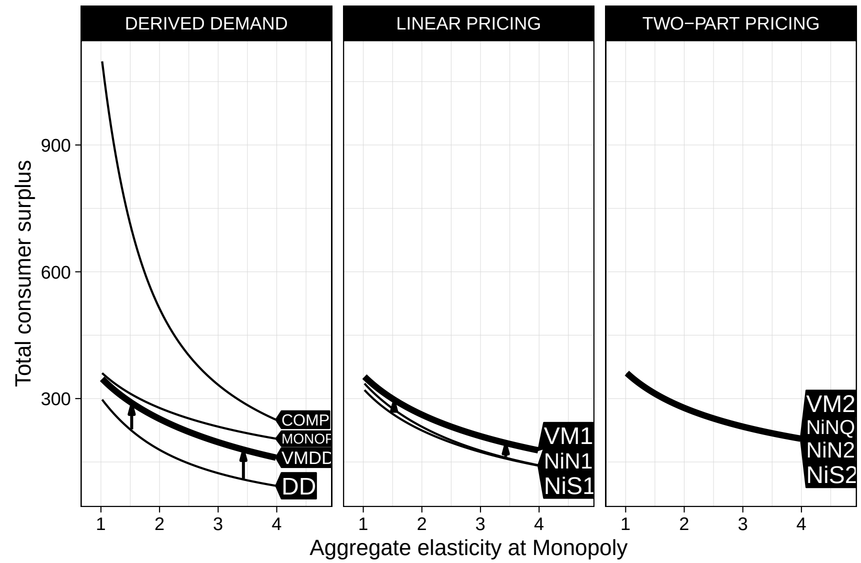


(d) Wholesale fees for firm A and B

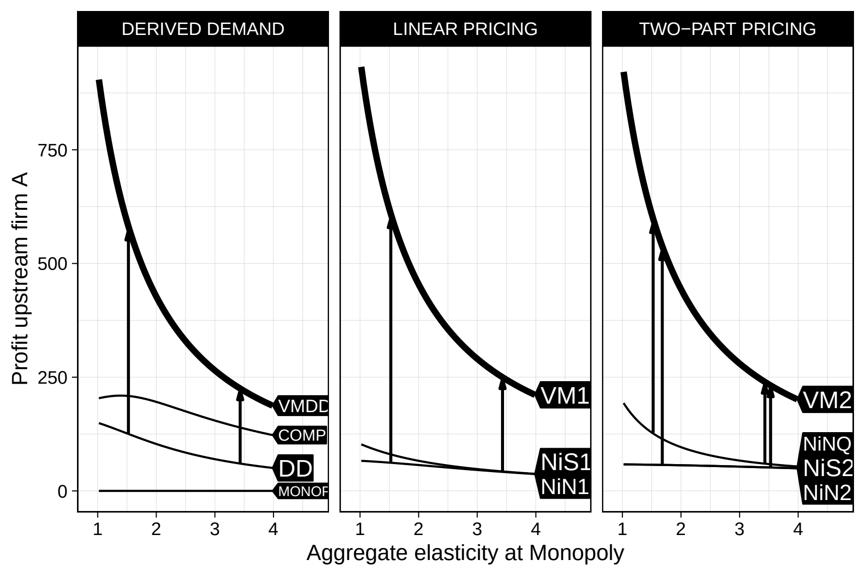
Figure 10: Wholesale prices and fees for a 2x1 setting: (a) shows the elimination of double marginalization for the vertically integrated firm; (b) shows the reducing of rival's revenue for the rival firm; (c) shows the decrease in total marginal cost for the rival firm as a result of reducing of rival's revenue; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing



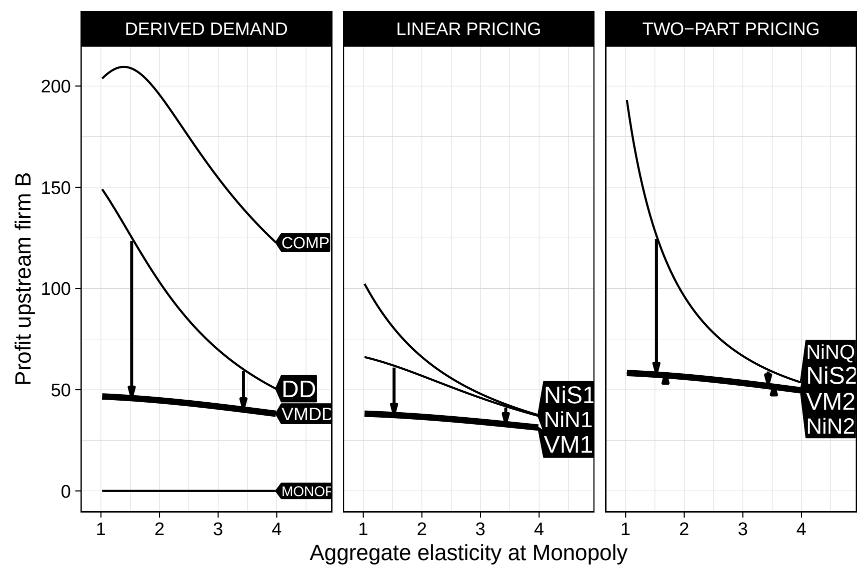
(a) Total quantity vs ae



(b) Consumer surplus vs ae



(c) Profit firm A and 1 vs ae



(d) Profit firm B vs ae

Figure 11: Profit and welfare in a 2x1 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the profit of firm A - post-merger profit is always higher than pre-merger profit; and, (d) shows the profit of firm B - post-merger profit is always lower than pre-merger profit