Review of Industrial Organization
https://doi.org/10.1007/s11151-026-10059-z

Capacity Collusion and Overcharge Estimation

Thomas Fagart[1] **· Willem Boshoff**[1]

Received: 2 June 2020 / Accepted: 7 May 2026

Abstract
Collusion may change price dynamics prior to and, especially, following a formal period of coordination. This paper presents a dynamic model of capacity choice under partial irreversibility to study the implications of a capacity cartel on price dynamics and damage estimates. We study both transitory pre- and post-cartel adjustment effects as well as a long-run effect and find that collusion damage continues after the cartel ends, and may persist indefinitely when demand remains constant. These effects imply biased estimates of cartel overcharge by both dummy-variable and forecasting methods, both of which rely on data during and after the collusion.

Keywords Capacity investment and disinvestment · Dynamic games · Markov perfect equilibrium · Real options · Collusion · Cartel overcharge

JEL Classification: D43 · L13 · L25

1 Introduction

While collusion usually involves agreeing to and monitoring prices or sales, some cartels rely on a different collusive device: firms' production capacity. In the US steel cartel from 2005 to 2007, the major US steel companies coordinated to reduce the overall industry capacity; they closed blast furnaces all over the country.[1] Another recent example is from the US airline market, in which the executives of the four

[1] See Case (2015), p. 12.

✉ Willem Boshoff
wimpie2@sun.ac.za

[1] Department of Economics, Stellenbosch University, Private Bag X1, 7599 Stellenbosch, South Africa

Published online: 15 July 2026

major airlines are alleged to have conspired to keep flight capacities unchanged between 2009 and 2015, with the consequence of a 13 % increase in the firm's profits.[2]

Unlike prices or sales, production capacities have a Markovian property as these are available long after the initial investment date. They also exhibit a hysteresis characteristic, as dismantling production capacity often does not cover the cost of capacity building.

This paper examines how capacity collusion impacts price dynamics during and especially after capacity collusion, and, subsequently, the damage estimation.

More precisely, we present a dynamic model of capacity investment under partial irreversibility. We consider an exogenous period of collusion as our focus is on the effect of collusion on capacity investment and damage estimation, rather than the incentives for collusion or the type of collusion. We then simulate the equilibrium path to study the effect of collusion on capacity and price and use the simulated data to study the implications for overcharge estimation.

Our model highlights three effects of collusion on price dynamics during and following a formal cartel period: First, due to adjustment costs, colluding firms do not reach the monopoly steady state instantly. Therefore the effects of collusion increase gradually over time. Second, subsequent to collusion and again due to adjustment costs, firms invest gradually to reach a non-cooperative steady state after the collapse of the cartel; therefore, the effects of collusion decrease gradually over time. Third, several paths for the post-collusion non-cooperative equilibrium emerge, given that a change in capacity during the cartel period alters the long-run equilibrium path of the industry. Depending on demand conditions, this third effect may not be transitory. Both the second and third effects suggest that collusion harms – relative to a non-collusive counter-factual world – may persist well beyond the end of a collusive agreement.

Even if cartelists are not liable for these long-run effects, these effects have implications for the proper estimation of cartel damages, including during the formal cartel period. Using bootstrapping simulations, we identified biases in the estimation of the cartel overcharge. These biases emerge for both the dummy-variable and forecast methodologies that are typically used in overcharge estimation, as both often rely on data from the collusive period or after the collusive period.

The remainder of the paper is structured as follows. Section 2 discusses the literature on cartel transitions and post-collusion behavior. Section 3 introduces a preliminary model to provide some insight that underlies the lingering effect of collusion. Section 4 presents the general model and characterizes the non-cooperative equilibria, while Sect. 5 discusses the various effects of capacity collusion on the price dynamics. Section 6 analyzes how these effects bias the standard estimation of cartel overcharge. All proofs are presented in appendix A.

[2] As of the date of writing this paper the lawsuit is still ongoing, however, two of the four defendants have agreed to settle. https://nypost.com/2023/09/13/delta-united-must-face-class-action-lawsuit-over-airfares/

2 Literature Review

The literature on dynamic investment behavior and capacity collusion is sparse. Paha (2017) adapts the model that was devised by Besanko and Doraszelski (2004) to simulate the investment behavior of firms when they collude in price, but not on capacity. In addition to his result on the effect of uncertainty on cartel formation, Paha presents counter-intuitive evidence that a low discount factor can impair collusion.

Feuerstein and Gersbach (2003) study grim trigger strategies in a duopoly model with linear demand and partially irreversible investment, and show that collusion in capacity is feasible, but at a lower discount factor compared to the case of fully reversible investment. Fagart (2022) analyzes collusion in capacity in a duopoly model with irreversible investment and shows that the irreversibility of investment creates a long-run incentive for deviation: The firm may deviate not only for immediate increases in its profit but also to gain a dominant position in the market. By investing in new capacity, a firm that deviates from a collusive agreement commits itself to a higher level of production in the future, which reduces the future incentive of competitors to invest, and decreases the possibility of punishment. Fagart (2022) shows that, under discount factors close to one, the long-term incentive prevails and an increase in the discount factor decreases the possibility of collusion. The irreversibility of investment therefore changes the effect of the discount factor.

This literature focuses mainly on the feasibility of collusion in capacity (or in collusion in price under capacity investment). Our paper presents a different approach: We focus on the effect of collusion with respect to capacity on price dynamics: during and after collusion.

Much of the traditional empirical literature on collusion and cartel damages – which focuses predominantly on price-based collusion – implicitly assume that collusive effects end when a cartel dies. This idea of regime switching between competitive and collusive states has its foundations in Porter (1983) analysis of price data for the Joint Executive Committee cartel. Indeed, much of the collusive-damages literature (see Section 6) assumes that the effect of a price-fixing cartel on prices can be determined if data for both a collusive and a subsequent non-cooperative period are available (see Rubinfeld (1985)).[3]

A significant literature has subsequently highlighted post-collusive price persistence and its implication for damage estimates. Several recent papers aim at offering alternative approaches to isolating an off-equilibrium period following the demise of a cartel from subsequent competitive periods. See Frank and Schliffke (2013) for an early attempt that deals with the German cement cartel. Boshoff and Van Jaarsveld (2019) – analyzing the South African cement cartel – suggest an explicit regime-switching model, in which the number of alternative price regimes are empirically determined and where transition probabilities (into or out of particular regimes) can be used to create a moderating dummy variable that captures the slow entry into, or exit from, collusion. Boswijk et al. (2019) advance alternative structural break tests

[3] The empirical literature may also rely on pre-collusive prices, although several papers also combine prices from both pre- and post-collusive periods to obtain competitive reference prices

for detecting material changes in price behavior, which, like the regime-switching models, need not coincide with the end (or beginning) of collusion.

Much of this literature views the 'lingering' price effects of collusion that follows the demise of a cartel as temporary and capable of being isolated from subsequent non-cooperative price behavior. When it comes to capacity collusion, we show below that there are conditions under which such lingering effects may well persist, with important practical implications for the choice of reference period when studying the effects of such collusion. Furthermore, we find a central role for the trajectory of demand in determining the persistence of price effects following the collapse of a capacity cartel: As persistent effects emerge under a non-cooperative (post-collusive) equilibrium and conditions of weak demand growth, they are to be distinguished from price persistence due to continuing tacit collusion as is suggested in the literature that deals with price-fixing cartels (Harrington, 2004). These differences, and their interaction with demand conditions, have important implications for the empirical analysis of capacity collusion and for post-collusion monitoring.

3 Preliminary Model

Consider a market with a homogenous product and two firms (*A* and *B*) that compete in capacity. At each period, production is determined by the number of facilities that are owned by the firms: K_t^A and K_t^B; we assume that the firms always produce at full capacity. The inverse demand function is then $P(K_t^A, K_t^B) = 21 - \left(K_t^A + K_t^B\right)$. Firms face no production cost. Time is discrete and firms face a discount factor $\delta \in]\frac{1}{2}, 1[$. At each time period, firms can invest in a new facility for a sunk cost *I* or destroy a facility and receive back a third of their investment. We restrict the number of facilities that can be owned by a firm to 5, 6 and 7.[4]

The program of the firm is then to maximize:

$$\Pi_i(K_{-1}^i, K_{-1}^{-i}) = \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^i, K_t^{-i}) - I \max(K_t^i - K_{t-1}^i, 0) + \frac{I}{3} \max(K_{t-1}^i - K_t^i, 0)\right] \quad (1)$$

where $\pi_i(K_t^i, K_t^{-i}) = P(K_t^i, K_t^{-i})K_t^i$. Assume that $(1-\delta)I \in]2, 3[$.[5] As simple as it is, this game has a Markovian structure, which is a key element for our results. The definition of the non-cooperative equilibrium in Markovian games can usually be a

[4] This assumption comes from the fact that the joint profit of the stage game is maximized for a total capacity of 10.5 and that the Nash equilibrium of the stage game without any investment cost is (7, 7). With a bit of additional work on the proof of proposition 1, this assumption can be released for a discrete choice of capacity. The main interest of the assumption is to keep this toy model tractable.

[5] The cost of investment of firm *I* should be high enough for the firm to invest in a sixth facility but not in a seventh if the competitor holds six facilities.

bit challenging, but our game exhibits a unique Markov perfect equilibrium when focusing on strategies that do not depend on the competitor's past capacity.[6]

Proposition 1 *The strategies :*

$$\breve{K}_t^i(K_{t-1}^i, K_{t-1}^{-i}) = \begin{cases} 6 if K_{t-1}^i \leq 6 \\ 7 otherwise \end{cases} \tag{2}$$

for $i = \{A, B\}$ *form the unique Markov perfect equilibrium formed by strategies that do not depend on their competitor state variable.*

We consider this Markov perfect equilibrium to be a non-cooperative one. There are then four steady states : Both firms own six facilities; both firms own seven facilities; and one firm owning seven facilities whereas its competitor owns six.

In that framework, other equilibria exist that rely on strategies that depend on the competitor's past capacity, which allow the firms to reach some cooperative outcomes. For instance, the grim trigger strategy which consists of returning to the equilibrium forever if one competitor deviates is a collusive equilibrium.

Proposition 2 *If* $I > \frac{3(8-7\delta)}{2(1-\delta)}$, *the strategies :*

$$\tilde{K}_t^i(K_{t-1}^i, K_{t-1}^{-i}) = \begin{cases} 5 if K_{t-1}^{-i} = 5 and K_{t-1}^{-i} = 5 or if t = 1 \\ \breve{K}_t^i(K_{t-1}^i, K_{t-1}^{-i}) otherwise \end{cases} \tag{3}$$

for $i = \{A, B\}$ *form a Markov perfect equilibrium.*

Once in place, there is no reason for collusion to collapse. In practice however, collusion breaks down frequently. This can be due to: an investigation by a competition authority; a shock in demand or costs; bad communication among cartel members; a change in management; an internal risk investigation inside one of the firms involved; or even a moral shift by one of the directors.

The purpose of this paper is to analyze the conduct of the market when collusion breaks. We focus on non-expected collusion breaks that are independent of the market variables (no demand or cost shock). To model this effect we consider an exogenous time, *T*, at which the cartel breaks and firms return to the non-cooperative equilibrium. Due to the Markovian structure of capacity investments, collusion affects the investment path after it ends.

This is easily visible in our simple model: During collusion, each firm owns five facilities. At time *T*, both firms invest according to the non-cooperative equilibrium that is given by (1), which results in each firm's owning six facilities. This is a steady state and firms keep their six facilities for the rest of the game. If the initial state of the game is seven facilities for each firm, then firms switch to five facilities during collusion, and to six facilities each at the end of collusion. The collusive period has therefore changed the steady state of the industry, reducing its total capacity. The

[6] In sub-game perfect games, the repetition of the stage game equilibrium (when it is unique) is usually considered to be the non-cooperative dynamic equilibrium. In Markovian games, the interactions between the stage games prevent us from using the same definition.

price evolves from seven during the pre-collusive path to eleven during the collusive period and to nine afterwards. In this case, collusion not only harms consumer surplus during the collusive period but also in the long-run.

This model relies on some strong assumptions in order to establish the intuition of the lingering effect of collusion. The following sections relax these assumptions and show that this mechanism of long-run damage is still present when firms can invest in facilities of variable size: when capacities are smooth; when the firms face more complex investment cost; and when demand evolves.

4 General Framework

4.1 The Model

Consider a market with *n* firms that compete in capacities. Let K_t^i be the capacity of firm *i* at time *t*, and let K_t be the vector of capacity of all of the firms in the market. Each firm starts with some initial capacity K_0^i. At each period, firms can decide to increase or reduce their production by buying or selling some capacity. Purchases are made at a linear price p^+, and sales at a (also linear) price p^- (with $p^+ > 0$, $p^- > 0$ to represent the scrap value of capacity). Firms also face a quadratic adjustment cost γ. The investment cost is then:

$$C(K_t^i - K_{t-1}^i) = \begin{cases} p^+\left(K_t^i - K_{t-1}^i\right) + \frac{\gamma}{2}\left(K_t^i - K_{t-1}^i\right)^2 & \text{if } K_t^i > K_{t-1}^i \\ p^-\left(K_t^i - K_{t-1}^i\right) + \frac{\gamma}{2}\left(K_t^i - K_{t-1}^i\right)^2 & \text{if } K_t^i < K_{t-1}^i \end{cases}. \quad (4)$$

This cost function combines two usual elements of the literature on dynamics competition and investment: the concave adjustment cost of Reynolds (1991), Figui res C. (2009) or Genc and Zaccour (2014); and the partial/total irreversibility that can been seen in Grenadier (2002); Back and Paulsen (2009); Boyer et al. (2012) or Huisman and Kort (2015) for instance.

Demand is linear with a dynamic trend, *v*, and a random shock, ε_t, with zero mean, independently and identically distributed.[7] A firm's production depends on its capacity. Without loss of generality, we assume that the marginal cost of production is zero. There is a common discount factor, δ, and the expected profit function of firm *i* is given by:

$$\Pi_i(K_0) = \sum_{t=1}^{+\infty} \delta^{t-1} E\left[\left(A + vt + \varepsilon_t - b\sum_{j=1}^{n} K_t^j\right) K_t^i - C(K_t^i - K_{t-1}^i)\right]. \quad (5)$$

We assume that firms start with the same initial capacity.

[7] As ε_t is a random shock and not a random walk, the value of ε_t has no effect on the distribution of ε_{t+1} , and the expected profit of firms depends only on the previous capacities.

Each firm maximizes its expected profit.[8] To determine the non-cooperative equilibrium of the game, we focus on Markov perfect equilibrium. Within this framework, the equilibrium depends only on the previous capacity of the firms. As the random shock is centered, the Bellman equation gives:

$$\Pi_i(K_{t-1}) = \max_{K_t^i}\left[\left(A + vt - b\sum_{j=1}^{n} K_t^j\right) K_t^i - C(K_t^i - K_{t-1}^i) + \delta\Pi_i(K_t)\right]. \quad (6)$$

4.2 Non-Cooperative Equilibrium

In the model above, the optimal capacity of firm *i* depends on the value function, which is itself determined by the dynamic equation that results from (6), once the firm's strategy is determined. To solve this issue, we assume that the value function has a quadratic form and then determine the optimal capacity choice, before we obtain the parameters of the profit function by solving the dynamic equation.[9]

Let $K^+(.)$ and $K^-(.)$ be respectively the investment and disinvestment capacity as defined by:

$$K^+(K_{t-1}) = \frac{A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\,\gamma\sum_{i=1}^{n} K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta} \quad (7)$$

and

$$K^-(K_{t-1}) = \frac{A + vt + \delta\left(\alpha^- + \mu t\right) - p^-}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\,\gamma\sum_{i=1}^{n} K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta} \quad (8)$$

where α^+, α^-, β, ζ, μ are the solution of (70), (72), (68), (69), (71) given in appendix A. These definitions permit us to characterize the equilibrium.

Proposition 3 *For every β, ζ, μ solution of (68), (69), (71), there exist a Markov perfect equilibrium which is:*

$$K_t^{*i} = \begin{cases} K^+(K_{t-1}) if K_{t-1}^i < \xi^+ \\ K_{t-1}^i if \xi^- \geq K_{t-1}^i \geq \xi^+ \\ K^-(K_{t-1}) if K_{t-1}^i > \xi^- \end{cases} \quad (9)$$

where α^+, α^-, ξ^+ and ξ^- are defined in appendix A in (70), (72), (83) and (84). Firms stay symmetric on the equilibrium path.

[8] To simplify the notation, we use the same notation for the profit and its expectation.

[9] Equation (6) also presents a discontinuity issue due to the difference between the marginal price of investment, p^+ and the price of disinvestment p^-. Step 2 of the proof of proposition 3 in appendix A presents a methodology to overcome this difficulty.

The main property of this equilibrium is that it depends on the past capacities of the industry. When capacities are small, firms invest at each time to capacity $K^+(K_{t-1})$ until they reach the level ξ^+. When capacities are high, firms disinvest at each time to capacity $K^-(K_{t-1})$ until they reach the level ξ^-. Between these two levels, firms do not invest and maintain their previous capacity. Consequently, different paths of equilibrium exist that depend on the initial capacity.

In Figure 1, the solid line represents the non-cooperative choice of capacity as a function of time and the associated price pattern, for a duopoly that starts with small (left-hand graph) or, alternatively large (right-hand graph), capacities.[10] The top dashed lines represent the non-cooperative steady state when marginal profitability equals the price of disinvestment; and the bottom dashed lines represents the steady state when the marginal profitability equals the price of investment.

In the absence of adjustment costs, the firms reach these levels at the first period of the game. Indeed, in this case there are no incentives to delay investment. With adjustment cost, a firm can reduce its costs by delaying investment.

The fact that the choice of capacities depends on the initial levels of capacity is due to the difference between the price of investment (buying capacity) and disinvestment (selling capacity). Investing firms are trying to reach a level of capacity such that their marginal profitability equals the cost of investment whereas disinvesting firm are trying to reach a level of capacity such that their marginal profitability equals the cost of disinvestment. As the cost of investment is higher than the disinvest-

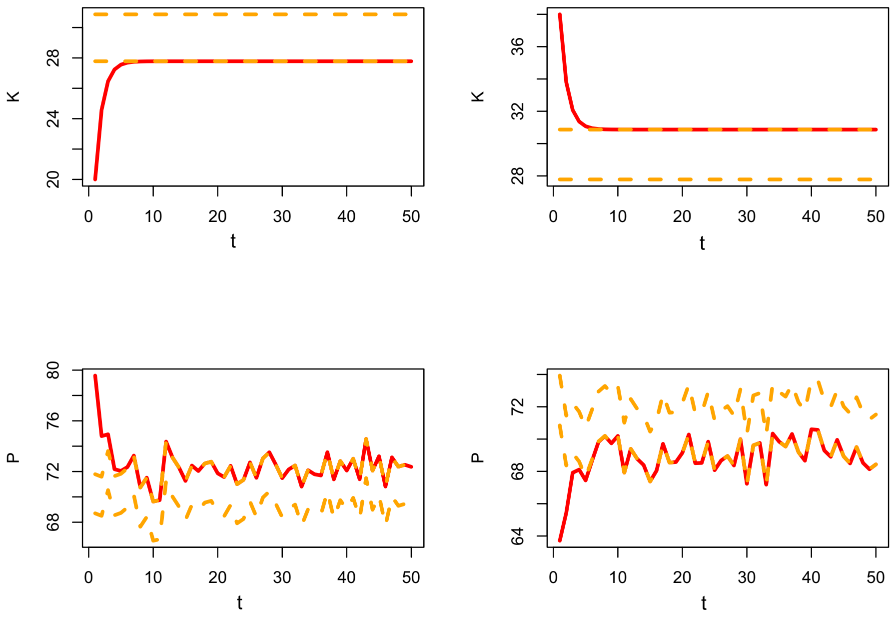

Fig. 1 Non-cooperative capacity path

[10]The parameter' values are $A = 100$, $b = 1$, $\delta = 0.9$, $c = 0.8$, $p^+ = 3$, $p^- = 0$. The two vectors of initial capacities are $K_1 = (43, 43)$ and $K_1 = (49, 49)$. The random shock follows a normal distribution of zero mean and variance 1.

ment cost, there is then a range of capacities for which the marginal profitability lies between these two costs.

The adjustment costs therefore determine the dynamics of investment, while the difference between the cost of investment and disinvestment is responsible for the multiplicity of non-cooperative equilibrium paths.

5 Collusion

We use the model to study collusion, which takes the form of an agreement between two dates, the starting date T_1 and the end date T_2. Crucially, we assume that none of these dates are anticipated, so the industry exhibits non-cooperative behavior immediately before T_1 and immediately after T_2. Between the date T_1 and T_2, we assume that firms maximize the joint industry profit and install the same, symmetric part of the monopoly capacity, as defined by (9) when $n = 1$.[11]

The irreversibility of investment generates three effects in the run-up to the cartel period and following the end of the cartel period: First, due to the adjustment cost, firms do not reach one of the monopoly steady states instantly. The effects of collusion therefore increase over time, as firms reduce their capacity step-by-step. Second, when the cartel terminates, firms also invest step-by-step to reach a new non-cooperative steady state. The effect of collusion therefore decreases with time. This effect is also due to the adjustment cost of investment. We term these two effects the transition effects. Third, the change in capacity during the cartel period may change the path of the non-cooperative equilibrium that is reached in the long-run. Indeed, reinvesting in capacity in a non-cooperative way may lead to an industry capacity that is inferior to the one before the collusive interlude, as the price of investment is higher than the price of disinvestment. We term this effect the lingering effect of collusion.

This section presents simulation results for capacity and price under increasing, decreasing and constant demand scenarios.[12] In each scenario, the collusive period starts at time period 20 and terminates at time period 30, with the entire sample period running from 0 to 50. To simulate the price pattern of the cartel, we assume that prices are subject to random shocks, following a Gaussian distribution.[13] Additional simulations that control for the existence of the lingering effect of collusion for different parameters' values can be found in Appendix B.

5.1 No Evolution of Demand

Suppose that demand remains constant over time. In this case, different initial capacities may lead to different steady states. Any capacity, such that marginal profitability

[11] As we noted above, we focus on the price dynamics of the cartel and do not study the feasibility of the cartel agreement. Feuerstein and Gersbach (2003) and Fagart (2022) study the possibility of collusion in capacity without adjustment cost.

[12] The parameter's values are $A = 100, b = 1, \delta = 0.9, c = 5, p^+ = 10, p^- = 0$. The demand evolution parameter takes different values in the three following cases: $v = \{0.1; 0; -0.1\}$. Two vectors of initial capacities are considered: $K_1 = (13.89, 13.89)$; and $K_1 = (15.43, 15.43)$.

[13] The mean of the Gaussian distribution is null, and its variance is 1.

is between the price of investment and the price of disinvestment, may be a steady state.

The top graphs of Fig 2 represent the capacity of the industry as a function of time, while the bottom graphs represent the market price as a function of time. In the left-hand graphs, initial capacities are low, which means that the marginal profitability of initial capacity is close to the price of investment; whereas in the right-hand graphs the initial capacities are high, which means that the marginal profitability of initial capacity is close to the cost of disinvestment. Such features help us to elucidate the dependence of capacity and price dynamics on the past by referring to the two extreme cases.

In the figure, the dashed lines represent the duopoly steady state when marginal profitability equals the price of disinvestment and when the marginal profitability equals the price of investment. Every non-cooperative steady state falls between these two lines. In the same way, the dotted-dashed lines represent the monopoly steady state when marginal profitability equals the price of disinvestment and when the marginal profitability equals the price of investment. The dotted lines represent the non-cooperative equilibrium, and the solid lines represent the collusive path of the industry.

At the beginning of the collusive period, the cartel reduces its capacity stepwise to reach the monopoly level. This represents the first transition effect. After the end of the collusive agreement, the industry also takes time to return to the non-cooperative long-run path. This is the second transition effect. These two transition effects are due to adjustment cost, as adjustment cost makes it profitable for the firm to delay a part of their investments.

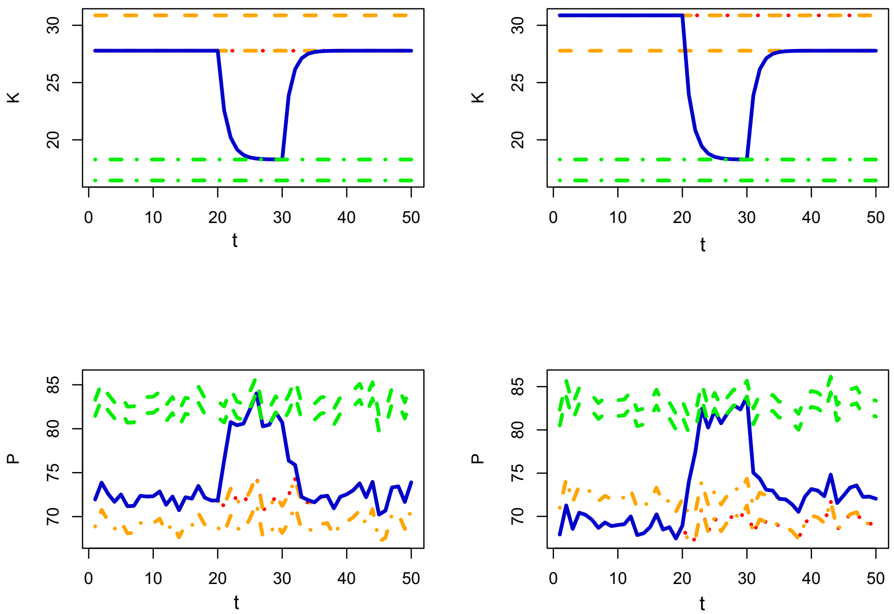

Fig. 2 Collusive path for the case of no demand evolution

In the left-hand graphs, capacities are low when the cartel starts (the marginal profitability of each unit of capacity equals the price of investment), and firms reach the same level of capacity after the end of the cartel and a transition phase. Indeed, after the cartel breaks, the firms are below capacity and seek to invest until the marginal profitability of capacity equals the price of investment. In this scenario, the non-cooperative path that is reached after collusion is equivalent to the initial non-cooperative path, due to the demand increase, and the lingering effect of collusion is therefore not present.

In contrast, in the right-hand graphs, capacities are high when the cartel starts: The marginal profitability of capacity equals the price of disinvestment. When the cartel breaks, firms are below capacity and will invest until the marginal profitability of capacity equals the cost of investment. As the cost of investment is higher than the cost of disinvestment, the long-run capacity level that is reached by the firms after the cartel breaks is inferior to the one before the cartel. Collusion has therefore changed the non-cooperative path that is followed by the firms: A higher price persists after the end of the cartel. This is the lingering effect of collusion.

5.2 Increasing Demand

Consider now that demand growth follows a positive trend. In this case, after some time, demand growth induces firms to invest regardless of the initial capacities. In the long-run every non-cooperative equilibrium follows the same pattern, such that the capacity ensures that the marginal profitability equals the price of investment. Figure 3 exhibits the capacity and price dynamics of the industry under non-cooperative and collusive equilibrium with high and low initial capacities, in a similar way to Fig 2.

As was seen above, the transition effects are present. Firms disinvest slowly to reach the monopoly long-run structure and invest step-by-step after the collusion ends. When initial capacities are low (as is seen in the left-hand graphs of Fig 3), the non-cooperative path is defined by the fact that the marginal profitability of capacity equals the cost of investment, and firms follow the same pattern after collusion; there is no lingering effect.

When initial capacities are large, the non-cooperative path for the firms is to not invest (nor disinvest) until the marginal profitability of capacity reaches the investment price. At the end of the cartel, if the marginal profitability of capacity in the initial non-cooperative path is still inferior to the investment price, a lingering effect may be present, as the new non-cooperative path reaches a marginal profitability of capacity that is equal to the price of investment. This lingering effect does not last forever, as every non-cooperative path converges in the long run.

5.3 Decreasing Demand

Assume now that demand follows a negative trend.[14] In this case, the non-cooperative equilibrium also follows a clear pattern in the long run: Regardless of the initial

[14] An extreme result may occur when demand becomes negative, as the associated capacity of firms and the market price is then zero.

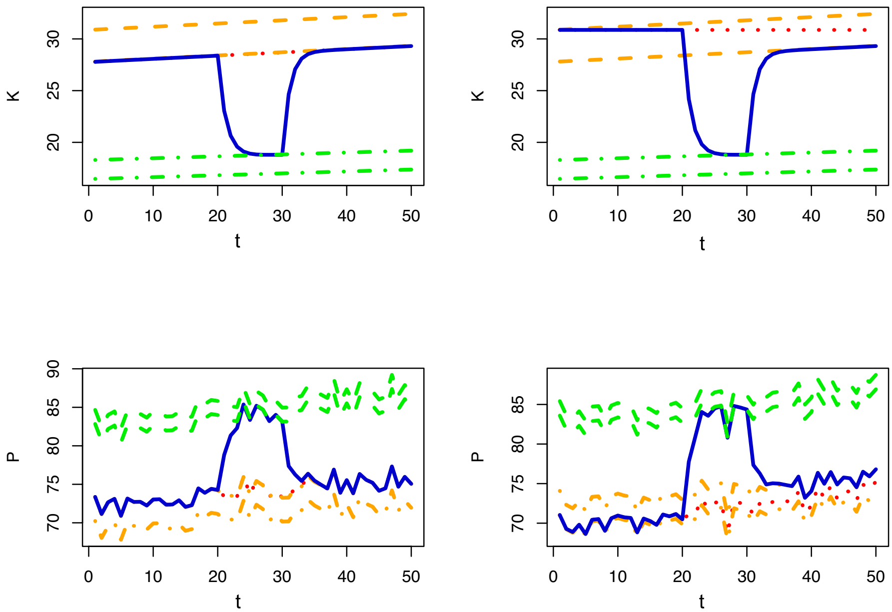

Fig. 3 Collusive path for the case of increasing demand

capacities, after a time firms will need to disinvest in order to respond to the demand reduction. In the long run, the marginal profitability of capacity equals the price of disinvestment. Figure 4 exhibits the capacity and price dynamics of the industry under non-cooperative and collusive equilibrium with high and low initial capacities, similar to Figs 2 and 3.

The lingering effect is present whatever the initial capacities, but does not last forever as in the case of no demand evolution. Indeed, when the firms invest to reach the non-cooperative long-run pattern after collusion, the demand continues to decrease. Firms stop investing when the marginal profitability of capacity equals the price of investment, but this marginal profitability continues to decrease with demand. After a while, the marginal profitability of collusion reaches the price of disinvestment, and firms start to disinvest again. At this point, firms catch up with the initial non-cooperative path and the collusion no longer has an impact on the market.

6 Bias Estimation of Collusive Overcharge

The previous section shows how the transition effects and the lingering effect may shape the price dynamics of capacity cartels. Price can be higher after the end of collusion than it would have been without collusion, and thus generate long-run damages. In the case of constant demand, the damage of the cartel may even persist indefinitely.

Follow-on damage cases focus on the cartel overcharge with the objective to compensate the consumers who bought the product at an inflated price: during the cartel

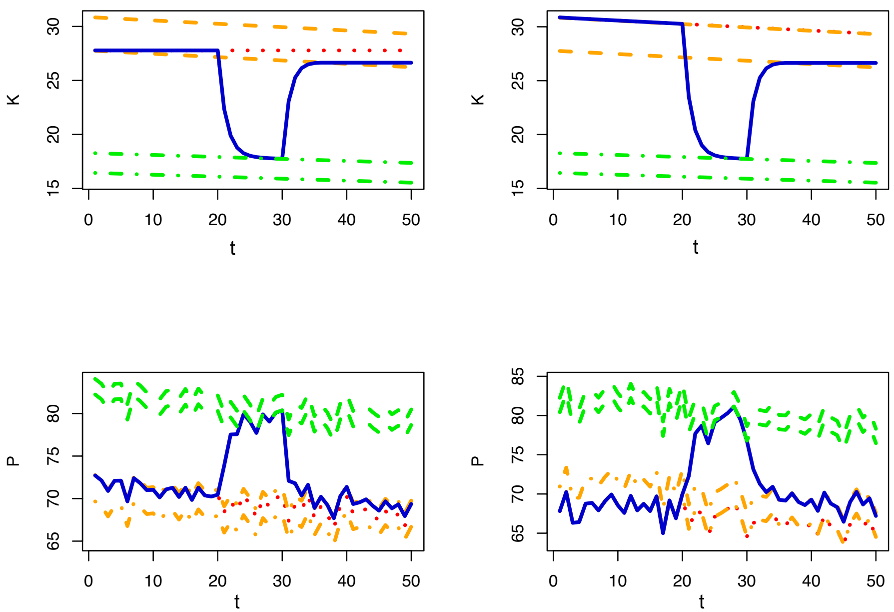

Fig. 4 Collusive path for the case of decreasing demand

period.[15] Our findings suggest that collusion in capacity may not only increase the market price after a cartel ends, but may also have consequences for the estimation of the overcharge during the cartel period.

To determine damages, the key unknown variable is the price that would have been realized in the absence of collusion (the ‘but for’ price); see White et al. (2006). The literature suggests several approaches to the estimation of a ‘but for’ price, which are highly dependent on the data that are available to the practitioner.[16] This section focuses on the forecast and the dummy-variable approaches, which are based only on the data of the relevant market and are used regularly in damages cases (Boshoff & Van Jaarsveld, 2019; schelrath et al., 2013; Rubinfeld, 1985).

The forecast method consists of estimating the (non-cooperative) price equation from data prior to, or following, the collusive period and then relying on this estimated equation to forecast the ‘but for’ price during the collusive period. Under the dummy-variable approach, the data during the collusion is also used when estimating the price equation, including a dummy variable of the cartel period in order to capture the collusive price overcharge.[17] The ‘but for’ price is then reconstructed by setting

[15] Courts do not typically account for consumers who did not buy the good because of the price increase. The legal damage of a cartel is therefore more restrictive than the notion of economic damage based on consumer surplus. See Rubinfeld (2012) for an overview of the legal approach.

[16] See Davis and Garcés (2009) p. 347-381, for a summary of the various alternative approaches.

[17] The dummy variable can also interact with demand or cost variables to take into account the effect of demand and cost shocks on the cartel overcharge; see Paha (2011) for an extensive discussion of the implications of such modelling choices. In our model, the evolution of demand and production cost are captured by a linear trend and our dummy-variable approach is therefore based on a simple dummy variable.

the dummy variable to be zero. Dummy-variable models have the advantage that they allow direct inference of whether prices during the collusive period are statistically different from non-collusive prices.

In the following sections we simulate the non-cooperative and the collusive price with the use of the theoretical model that was developed above, and apply the forecast and dummy-variable methods on the simulated data to predict a ‘but for’ price that we compare to the non-cooperative price previously simulated.

6.1 Biases Coming From the Transition Effects

In order to analyze the transition effects on the estimation of ‘but for’ prices, we simulate a scenario similar to the one that was presented in the left-hand graph of Fig 2, with constant demand and initial capacities so that their marginal profitability equals the price of investment.[18]

Figure 5 presents the ‘but for’ prices that are estimated with the different approaches. The collusive price is represented by the solid line; the non-collusive price is represented by the dotted line (in plain line); and the estimated ‘but for’ price is dashed line. The forecasting method is used in the top graphs with, going from left to right: pre-collusive data; post-collusive data; and pre- and post-collusive data. The dummy-variable method is used in the bottom graph with, from left to right: pre- and during collusion data; during and post-collusive data; and pre-, during, and post-collusive data.

If the forecast method with pre-collusive data offers a good prediction of the non-cooperative price during the collusion, the other approaches seem to be biased. The post-cartel transition effect artificially increases the price just after collusion and biases the post-collusive data. The slope of the forecast estimation is then reduced, and the overcharge of the cartel is underestimated. The effect of the post-cartel transition effect is more subtle when using pre- and post-collusive data with the forecast method, which slightly affects the slope and the level of the ‘but for’ prices.

With the dummy-variable approach, the within-cartel transition effect creates another bias. When the pre- and during collusion data are used, this increases the slope and level of the ‘but for’ price, which results in a reduction of the overcharge estimation. When the during and post-collusion data are used, both transition effects are present, which leads to an increase in the slope of the ‘but for’, which may contribute to an overestimation of the cartel overcharge. The utilization of pre-collusive data reduces this bias.

Figure 5 is based on the result of a single simulation, which may be misleading, given the relatively small number of data points (20 points before the collusion, 10 during collusion, and 20 after). To verify that the patterns that are portrayed by Fig 5, Table 1 reports the difference in percentage between the mean overcharge during the cartel period and the ’but for’ price that is predicted by each of the six methods of estimation that are considered.[19]

[18] We set the initial capacities so that their marginal profitability equals exactly the price of investment in order to not create a bias in pre-collusive data; therefore the values of the initial capacities in Table 1 change.

[19] The results presented are the average of this difference over 10,000 simulations.

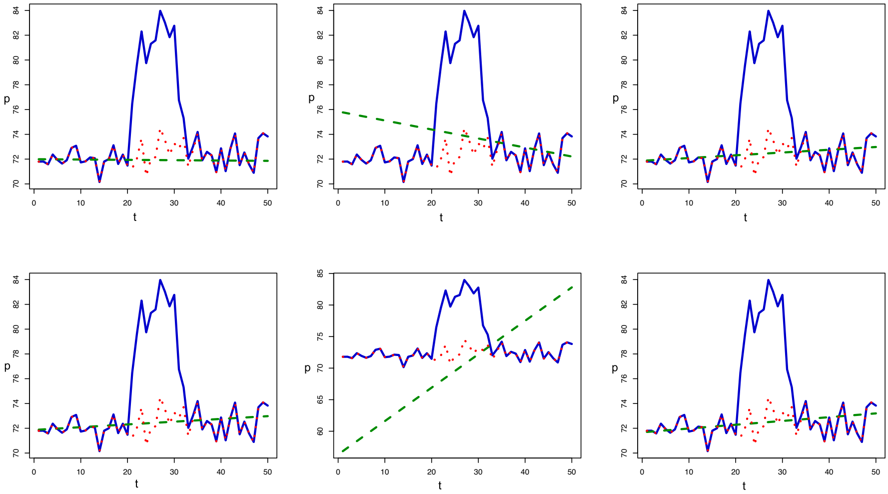

Fig. 5 Estimated 'but for' prices without the lingering effect of collusion

Table 1 Biases in the estimated cartel overcharge without a lingering effect

c	$(F1)$	$(D1)$	$(F2)$	$(D2)$	$(F3)$	$(D2)$
4	99.79	80.53	79.65	137.79	97.90	97.84
4.5	100.03	80.86	80.24	137.90	98.00	97.94
5	100.01	81.07	80.99	138.13	98.09	98.10
5.5	99.97	81.33	81.91	138.15	98.20	98.14
6	100.05	81.60	83.00	138.31	98.30	98.22

The first column in Table 1 indicates the various quadratic cost parameters that are considered.[20] Columns $(F1)$ and $(D1)$ present the estimated overcharge using, respectively, the forecast with pre-collusive data and the dummy-variable methods with pre- and during collusion data. Columns $(F2)$ and $(D2)$ relate to the forecast method using post-collusive data and the dummy-variable methods with during and post-collusive data. Columns $(F3)$ and $(D3)$ indicate the estimated overcharge based on the forecast method with pre- and post-collusive data and the dummy-variable methods with pre-, during and post-collusive data.

Table 1 confirms the conclusion from Fig 5 and the existence of bias due to the transition effect of collusion. This bias may lead to a 20 percent reduction of the cartel overcharge in the case of the dummy-variable estimation using pre- and during collusion data, or in the case of using forecasting with post-collusive data. The utilization of the dummy-variable estimation with during and post-collusive data leads to an around 40 percent over-estimation of the overcharge. Such levels are not negligible in antitrust cases.

The utilization of pre- and post-collusive data significantly reduces the bias, but does not fully suppress it.

[20] The other parameters are similar to those that were presented in footnote 10.

Moreover, Table 1 shows that an increase in the quadratic cost parameter, which implies higher transition effects, increases the estimation of the cartel overcharge, which leads to a more accurate estimation in almost every case except when using the dummy-variable approach with during and post-collusive data (and the forecasting approach based only on pre-collusive data).

6.2 Biases From the Lingering Effect

To study the consequences of the lingering effect on the estimation of the cartel overcharge, this sub-section assumes that the marginal profitability of the initial capacities equals the cost of disinvestment, which leads to a path of investment that is similar to the one presented in the right-hand graph of Fig 2. Figure 6 shows the 'but for' prices that are estimated with different approaches, in a similar way to Fig 5.

The lingering effect of collusion leads to post-collusive prices that are higher than the non-cooperative benchmark. This results in an overestimation of the but-for price when post-collusive data are utilized, and in a lower estimation of the cartel overcharge compared to the situation without a lingering effect, which can be confirmed in Table 2 below. In the case of the dummy-variable approach with only during- and post-collusion data, this reduces the overestimation of the cartel overcharge, but does not remove it. In the other cases, the underestimations have been worsened.

Table 2 gives the difference in percentages between the mean overcharge during the cartel period and the 'but for' price that is predicted with one of the six methods of estimation that were considered.[21] Table 2 confirms the indication that is given by the simulations that are presented in Fig 6: The utilization of post-collusive data reduces the estimation of the 'but for' prices, which leads to a lower estimation of the cartel

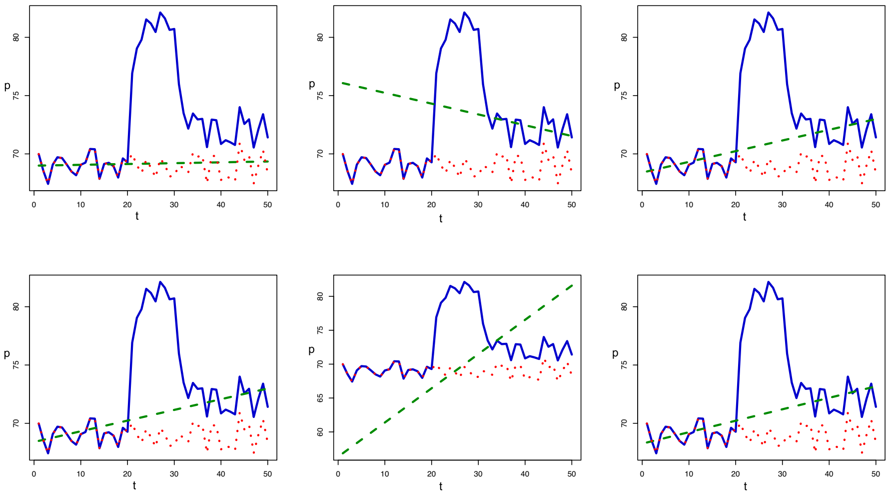

Fig. 6 'But for' estimations with a lingering effect of collusion

[21] The results presented are the average of this difference over 10,000 simulations.

Table 2 Biases in estimated cartel overcharge with a lingering effect

c	$(F1)$	$(D1)$	$(F2)$	$(D2)$	$(F3)$	$(D2)$
4	100.00	80.79	58.31	102.60	85.35	85.12
4.5	100.02	80.907	58.61	102.22	85.30	85.01
5	100.07	81.18	59.01	102.01	85.23	84.96
5.5	100.00	81.48	59.40	101.65	85.12	84.89
6	99.99	81.63	59.74	101.31	85.01	84.77

overcharge. This may lead to a 40 percent underestimation of the cartel overcharge when using the forecasting approach with post-collusive data.

The bias that is associated with the lingering effect remains important even when pre- and post-collusive data are used.

7 Concluding Remarks and Discussion

This work studies the price dynamics of collusion in capacity and its implications for cartel damage estimates. We simulate the behavior of an oligopoly that compete in capacity, with partial irreversible investment and adjustment cost. Due to the hysteresis characteristic of investment, several non-cooperative equilibrium paths emerge depending on the initial capacities. When the industry faces a collusive interlude between two exogenous dates, this creates three effects on price behavior.

Due to the adjustment cost of investment, firms reduce or increase their capacity step-by-step, which leads to two transition effects: When collusion starts, it takes time for price to reach the monopoly level. When the collusion ends, firms take time to increase their capacity, and the price also takes time to reduce to a non-cooperative level. We also show the existence of a long-run effect, the lingering effect, when the post-collusive non-cooperative equilibrium path differs from the pre-collusive non-cooperative equilibrium path. In the absence of changes in demand, collusion may lead to a permanent price increase after the end date of the cartel.

One of the hypotheses of our model is that the cartel start and end dates are exogenous. This may reflect unanticipated random events that affect the beginning or the end of collusion, such as: a change in management; a fortuitous private meeting between directors; or a hardening of the antitrust legislation. When an unanticipated random event influences the investment pattern, as a random shock, or a technological breakthrough, our view is that the identified effects may be overshadowed by the change in the investment pattern. When the event can be anticipated, capacity investments may be influenced not only by the event, but also by the change in competition that it may imply. Intuitively, a firm that anticipates the beginning of collusion may delay its investments, so as to not have to disinvest a freshly installed capacity unit. A firm that anticipates the end of a cartel may invest before the end date or accumulate some financial reserve to prepare for the investment phase that will follow.

Our model rests on the assumption that firms produce at full capacity and that collusion rests on capacity. These two assumptions are related to each other, as collusion in price with production at full capacity will lead to similar results as our model: A change in price implies a change in quantity that requires a change in the production capacity. In reality, firms can always produce under their capacity; but keeping unused capacities has a cost as firms need to pay for workers and maintenance.

Could our results apply in the case of price collusion in a market with flexible capacity? This question clearly calls for more research; however, the initial supposition is that even if capacity is not the collusive device, the reduction in sales that is required to increase the price reduces the value of capacity for the firm. Our results may then still hold, as long as the hysteresis of capacity investment and the adjustment cost are sufficiently high in the industry.

The investment behavior of the South African cement market support this view. Sharing information on sales, South African cement manufacturers operated an illegal cartel that started in 1998, collapsed in 2006, re-formed in 2008 and was finally dismantled in 2011. Fig 7 presents the capacity investment decisions from the largest firm in the cartel based on the value of property, plant, and equipment in its annual financial reports. Fig 7 shows that capital investment was quite stable during periods of collusion, but accelerated both during a short price-war period between 2006-2008 and following the demise of the cartel in 2011.

The three effects that are identified in this work affect the estimation of the cartel overcharge during the collusion period. Focusing on the forecast and the dummy-variable method, we establish that every estimation that is based on data during or after the collusion is biased. This leads to an underestimation of cartel overcharge in every case that is based on collusive and post-collusive data, except in the dummy-

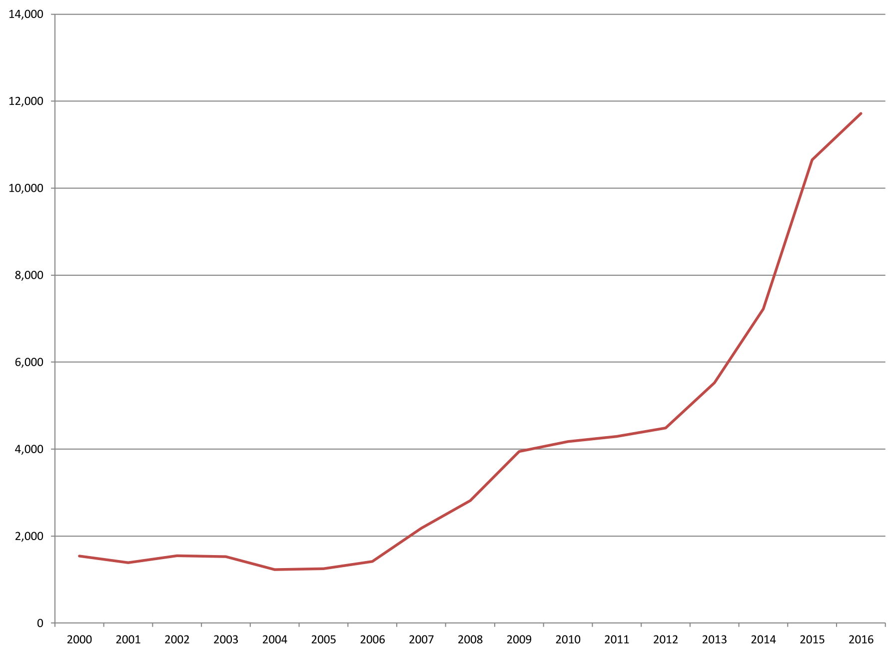

Fig. 7 Capacity evolution of pretoria portland cement company

variable approach which overestimates the cartel overcharge. These biases are not negligible, and simulation shows that they can lead to under- or over-estimation of the overcharge by as much as 40 per cent.

This result raises two econometric questions: how to test for the presence of the transition or lingering effect in the data; and how to correct the identified bias. The use of econometric techniques that are based on a comparable market - yardstick method - or the difference in differences method may provide an interesting lead, depending on the characteristics of the comparable market. The use of capacity data, when available, may be another compelling direction.

Finally, this work shows that the damage that is wrought by a capacity cartel can continue after it has ended, which gives an economic foundation for practitioners to consider post-collusive behavior when evaluating the gravity of a cartel offense.

Appendix A

See Tables 3, 4, 5 and 6

Table 3 Transition and lingering effects of variation of c

c	T1	T2	T3
3.5	4.071075	3.647549	2.689297
4	4.106737	3.807136	2.711971
4.5	4.244402	3.754635	2.884490
5	4.238514	4.023714	3.081925
5.5	4.298018	3.930207	3.219007
6	4.362104	4.031253	3.531825
6.5	4.350753	4.022934	3.722224

Table 4 Transition and lingering effects of variation of p^+

p^+	T1	T2	T3
7	3.872529	3.456232	3.962924
8	3.999508	3.704868	3.650477
9	4.122996	3.781792	3.329050
10	4.268265	3.890444	3.144519
11	4.481792	4.016530	2.761867
12	4.529839	4.168512	2.415551
13	4.757189	4.340566	2.078575

Table 5 Transition and lingering effects of variation of *A*

A	T1	T2	T3
70	5.998965	5.464766	3.043879
80	5.503537	5.025452	3.134184
90	4.783617	4.467002	3.055648
100	4.349388	3.948343	3.063506
110	3.679456	3.418267	3.027397
120	3.077156	2.871353	3.062758
130	2.664073	2.420685	3.081397

Table 6 Transition and lingering effects of variation of *b*

b	T1	T2	T3
0.7	4.811691	3.860455	4.91023723
0.8	4.645426	3.900769	4.26856712
0.9	4.494778	3.914975	3.72320420
1	4.304259	3.857947	3.13480121
1.1	4.318966	3.915037	2.37151643
1.2	4.224332	4.003861	1.29026541
1.3	4.340916	4.052152	0.03751716

Proof of Proposition 1

Let us start by establishing that the matrix of stage game profits $\left(\pi_i(K_t^i, K_t^{-i}), \pi_{-i}(K_t^i, K_t^{-i})\right)$ for the different feasible sets of facilities' numbers are:

$$\begin{array}{cccc} & K_t^i = 5 & K_t^i = 6 & K_t^i = 7 \\ K_t^{-i} = 5 & (55, 55) & (60, 50) & (63, 45) \\ K_t^{-i} = 6 & (50, 60) & (54, 54) & (56, 48) \\ K_t^{-i} = 7 & (45, 63) & (48, 56) & (49, 49) \end{array}$$

Assume that the strategy of firm $-i$ does not depend on firm's i capacity. The path of capacity $\left(K_t^{-i}\right)_{t>0}$ is independent of $\left(K_t^i\right)_{t>0}$ and the best response of firm *i* maximizes the dynamic programming problem:

$$\left(K_t^{BR_i}\right)_{t>0} = \arg \max_{K_t^i, t>0} \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^i, K_t^{-i}) - I \max(K_t^i - K_{t-1}^i, 0) + \alpha I \max(K_{t-1}^i - K_t^i, 0)\right]. \quad (10)$$

The rest of the proof is composed of two steps. In the first step, we show that a firm's best response capacity to a strategy that does not depend on the competitor's past capacity never equals 5. In the second step, we show that, assuming that the capacity of firm $-i$ is not equal to 5, the best response of firm *i* is the strategy defined as (2).

Step 1

Assume that there exists a $s > 0$ such as $K_s^{BR_i} = 5$ and $K_{s+1}^{BR_i} > 5$. Consider the alternative strategy $\left(K_t^{Alt_i}\right)_{t>0}$ so that $K_t^{Alt_i} = K_t^{BR_i}$ for all $t \neq s$ and $K_s^{Alt_i} = 6$. Then the difference between the value functions of the two capacity paths, Δ, is given by:

$$\Delta = \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^{BR_i}, K_t^{-i}) - I * \max(K_t^{BR_i} - K_{t-1}^{BR_i}, 0) + \alpha I \max(K_{t-1}^{BR_i} - K_t^{BR_i}, 0) \right]$$
$$- \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^{Alt_i}, K_t^{-i}) - I * \max(K_t^{Alt_i} - K_{t-1}^{Alt_i}, 0) + \alpha I \max(K_{t-1}^{Alt_i} - K_t^{Alt_i}, 0) \right].$$

We have:

$$\begin{aligned}
&\Delta = \pi_i(5, K_s^{-i}) + \alpha I \max(K_{s-1}^{BR_i} - 5, 0) - \delta I * \max(K_{s+1}^{BR_i} - 5, 0) \\
&- \pi_i(6, K_s^{-i}) + I * \max(6 - K_{s-1}^{BR_i}, 0) - \alpha I \max(K_{s-1}^{BR_i} - 6, 0) \\
&+ \delta I * \max(K_{s+1}^{BR_i} - 6, 0) - \alpha \delta I \max(6 - K_{s-1}^{BR_i}, 0),
\end{aligned}$$

which gives:

$$\Delta = \pi_i(5, K_s^{-i}) + \alpha I \max(K_{s-1}^{BR_i} - 5, 0) - \alpha I \max(K_{s-1}^{BR_i} - 6, 0) - \pi_i(6, K_s^{-i}) + I * \max(6 - K_{s-1}^{BR_i}, 0) - \delta I, \tag{11}$$

as $K_{s+1}^{BR_i} > 5$. Then there are two possibilities. If $K_{s-1}^{BR_i} > 5$,

$$\Delta = \pi_i(5, K_s^{-i}) + \alpha I - \pi_i(6, K_s^{-i}) - \delta I < 0, \tag{12}$$

as $\alpha < \delta$ and if $K_{s-1}^{BR_i} = 5$

$$\Delta = \pi_i(5, K_s^{-i}) - \pi_i(6, K_s^{-i}) + (1 - \delta) I < 0, \tag{13}$$

as $(1 - \delta)I < 3$. Therefore, the alternative strategy gives a better payoff than the best response strategy, which contradicts the definition of best responses and implies that there is no $s > 0$ such that $K_s^{BR_i} = 5$ and $K_{s+1}^{BR_i} > 5$.

Assume now that there is a s such that $K_t^{BR_i} = 5$ for all $t > s$, and considers the alternative strategy $\left(K_t^{Alt_i}\right)_{t>0}$ such that $K_t^{Alt_i} = K_t^{BR_i}$ for all $t < s$ and $K_s^{Alt_i} = 6$ for $t \geq s$. Then the difference of the value functions of the two capacity paths, Δ, gives:

$$\Delta = \delta^s \left(\sum_{t=s}^{+\infty} \delta^{t-1} \left[\pi_i(5, K_t^{-i}) - \pi_i(6, K_t^{-i}) \right] \right) - \delta^s I * \max(5 - K_{s-1}^{BR_i}, 0) + \delta^s I * \max(6 - K_{s-1}^{BR_i}, 0), \tag{14}$$

so

$$\Delta = \frac{\delta^s \left(\pi_i(5, K_s^{-i}) - \pi_i(6, K_s^{-i}) + (1 - \delta) I \left[\max(6 - K_{s-1}^{BR_i}, 0) - \max(5 - K_{s-1}^{BR_i}, 0) \right] \right)}{1 - \delta} < 0 \tag{15}$$

as $(1 - \delta)I < 3$. Therefore, the alternative strategy gives a better payoff than the best response strategy, which implies that there is no $s > 0$ such that $K_t^{BR_i} = 5$ for all $t > s$. There is then no $s > 0$ such that $K_s^{BR_i} = 5$.

Therefore, a firm's best response capacity to a strategy that does not depend on its competitor's past capacity never equals 5.

Step 2

Assume now that the strategy of firm $-i$ is never to own a capacity that is equal to 5: that $K_t^{-i} \neq 5$ for all $t > 0$.

Assume that there exists one $s > 0$ such that $K_s^{BR_i} = 7$ and $K_{s+1}^{BR_i} = 6$. Let $\left(K_t^{Alt_i}\right)_{t>0}$ be an alternative strategy defined by $K_t^{Alt_i} = K_t^{BR_i}$ for all $t \neq s+1$ and $K_{s+1}^{Alt_i} = 7$. The difference of the value functions of the best response capacity path and the alternative capacity path -Δ- is then:

$$\Delta = \delta^s \left[\pi_i(6, K_{s+1}^{-i}) + \alpha I - \pi_i(7, K_{s+1}^{-i})\right] \\ + \delta^{s+1} \left[I \max(K_{s+2}^{Alt_i} - 7, 0) - I \max(K_{s+2}^{BR_i} - 6, 0) - \alpha I \max(7 - K_{s+2}^{Alt_i}, 0) + \alpha I \max(6 - K_{s+2}^{BR_i}, 0)\right].$$

If $K_{s+2}^{BR_i} = 7$

$$\Delta = \delta^s \left[\pi_i(6, K_{s+1}^{-i}) + \alpha I - \pi_i(7, K_{s+1}^{-i})\right] - \delta^{s+1} I < 0 \tag{16}$$

as $\alpha < \delta$ and if $K_{s+2}^{BR_i} \leq 6$, then:

$$\Delta = \delta^s \left[\pi_i(6, K_{s+1}^{-i}) - \pi_i(7, K_{s+1}^{-i}) + \alpha\,(1 - \delta)\, I\right] < 0 \tag{17}$$

as $\alpha\,(1-\delta)\, I < 1$. This is impossible as $\left(K_t^{BR_i}\right)_{t>0}$ is supposed to be the best response of firm i. Therefore, for all $s > 0$ if $K_s^{BR_i} = 7$, then $K_{s+1}^{BR_i} \neq 6$. The same reasoning applies if there is one $s > 0$, $K_s^{BR_i} = 7$ and $K_{s+1}^{BR_i} = 5$ or if $K_s^{BR_i} = 6$ and $K_{s+1}^{BR_i} = 5$.

Assume now that there exists one $s > 0$ such that $K_s^{BR_i} = 6$ and $K_{s+1}^{BR_i} = 7$. Let $\left(K_t^{Alt_i}\right)_{t>0}$ be an alternative strategy defined by $K_t^{Alt_i} = K_t^{BR_i}$ for all $t \neq s+1$ and $K_{s+1}^{Alt_i} = 6$. The difference of the value functions of the two capacity paths -Δ- is:

$$\Delta = \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^{BR_i}, K_t^{-i}) - I * \max(K_t^{BR_i} - K_{t-1}^{BR_i}, 0) + \alpha I \max(K_{t-1}^{BR_i} - K_t^{BR_i}, 0)\right] \\ - \sum_{t=1}^{+\infty} \delta^{t-1} \left[\pi_i(K_t^{Alt_i}, K_t^{-i}) - I * \max(K_t^{Alt_i} - K_{t-1}^{Alt_i}, 0) + \alpha I \max(K_{t-1}^{Alt_i} - K_t^{Alt_i}, 0)\right].$$

$$\Delta = \delta^s \left[\pi_i(7, K_{s+1}^{-i}) - I - \pi_i(6, K_{s+1}^{-i})\right] \\ + \delta^{s+1} \left[I \max(K_{s+2}^{BR_i} - 7, 0) - I \max(K_{s+2}^{BR_i} - 6, 0) - \alpha I \max(7 - K_{s+2}^{BR_i}, 0) + \alpha I \max(6 - K_{s+2}^{BR_i}, 0)\right].$$

If $K_{s+2}^{BR_i} = 7$,

$$\Delta = \delta^s \left[\pi_i(7, K_{s+1}^{-i}) - (1 - \delta)\, I - \pi_i(6, K_{s+1}^{-i})\right] < 0 \tag{18}$$

as $(1 - \delta)\, I > 2$, and if $K_{s+2}^{BR_i} \leq 6$

$$\Delta = \delta^s \left[\pi_i(7, K_{s+1}^{-i}) - I + \alpha\delta I - \pi_i(6, K_{s+1}^{-i})\right] < 0 \tag{19}$$

as $\alpha < 1$. $\Delta < 0$ contradicts the fact that $\left(K_t^{BR_i}\right)_{t>0}$ is the best response of firm *i*. This implies that if $K_s^{BR_i} = 6$, $K_{s+1}^{BR_i} \neq 7$.

The best response of firm *i* then verifies $K_{t+1}^{BR_i} = \begin{cases} 6 \text{ if } K_t^{BR_i} \leq 6 \\ 7 \text{ if } K_t^{BR_i} = 7 \end{cases}$. ■

Proof of Proposition 2

To ensure that the collusion is feasible, no competitor should have a better payoff by deviating. There are two possible deviations, as a deviating firm can invest to reach six or seven facilities.

When the deviating firm installs a sixth facility, deviation is profitable when the profit of the deviating period plus the profit of the return to the non-cooperative equilibrium is higher than the profit from collusion:

$$60 - I + \frac{\delta}{1-\delta}54 > \frac{55}{1-\delta}, \tag{20}$$

which yields:

$$\delta\left(I - 60 + 54\right) > I - 60 + 55, \tag{21}$$

which can be rewritten as:

$$\delta > \frac{I-5}{I-6}. \tag{22}$$

(22) contradicts $\delta < 1$, so deviating to a sixth facility is never profitable.

When the deviating firm installs a sixth and a seventh facility, deviation is profitable when the profit of the deviating period plus the profit of the return to the non-cooperative equilibrium is higher than the profit from collusion:

$$63 - 2I + \frac{\delta}{1-\delta}56 > \frac{55}{1-\delta}, \tag{23}$$

which yields:

$$\frac{63\left(1-\delta\right) - 55 + \delta 56}{1-\delta} > 2I, \tag{24}$$

which can be rewritten as:

$$I < \frac{8-7\delta}{1-\delta}. \tag{25}$$

(25) contradicts $I > \frac{3(8-7\delta)}{2(1-\delta)}$, so deviating to a sixth and seventh facility is never profitable.

The previous part has then shown that once in the collusive equilibrium, firms do not deviate from the equilibrium. The following will demonstrate that the collusive equilibrium can be implemented even if firms have an initial number of facilities larger than five. If a firm initially has six or seven facilities, it may deviate from the collusive equilibrium by simply keeping its initial facilities.

If a firm initially owns six facilities and deviates from the collusive investment by not divesting, its competitor follow the non-cooperative path after the deviation and invest in a sixth facility. The deviation is therefore profitable if:

$$60 + \frac{\delta}{1-\delta}54 > \frac{55}{1-\delta} + \frac{I}{3}, \tag{26}$$

which gives:

$$(1-\delta)\, I < 3\left((1-\delta)\, 60 + \delta 54 - 55\right), \tag{27}$$

which can be rewritten as:

$$I > \frac{3\,(5-6\delta)}{1-\delta}. \tag{28}$$

When $\delta > \frac{1}{2}$, $\frac{3(8-7\delta)}{2(1-\delta)} > \frac{3(5-6\delta)}{(1-\delta)}$ as $(8-7\delta) > 2\,(5-6\delta)$, and (28) is verified as $I > \frac{3(8-7\delta)}{2(1-\delta)}$.

If a firm initially owns seven facilities and deviates from the collusive investment by not divesting, its competitor will follow the non-cooperative path after the deviation and invest in a sixth facility. The deviation is therefore profitable if:

$$63 + \frac{\delta 56}{1-\delta} > \frac{55}{1-\delta} + 2\alpha I, \tag{29}$$

which gives:

$$(1-\delta)\, I > \frac{(1-\delta)\, 63 - 55 + 56\delta}{2\alpha}, \tag{30}$$

which can be rewritten as:

$$(1-\delta)\, I > \frac{8-7\delta}{2\alpha}. \tag{31}$$

■

Proof of Proposition 3

The capacity strategies, $\left(K^{*i}\right)_{\text{for all } i}$, which are defined by (9) form a Markov perfect equilibrium, if there exists value functions, $\left(\Pi_i^*(K_t)\right)_{\text{for all } i}$, such that for each i, K^{*i} maximizes the inter-temporal profit of firm i:

$$K^{*i}(K_{t-1}) = \arg\max_{K_t^i}\left[\left(\left(A + vt - b\left(K_t^i + \sum_{\substack{j=1\\ j\neq i}}^{n} K^{*j}(K_{t-1})\right)\right)\right) K_t^i - C(K_t^i - K_{t-1}^i) + \delta\Pi_i^*(K_t)\right], \quad (32)$$

and Π_i^* verifies the dynamics equation of profit:

$$\Pi_i^*\left(K_{t-1}\right) = \left[\left(A + vt - b\sum_{j=1}^{n} K_t^{*j}\right) K_t^{*i} - C(K_t^{*i} - K_{t-1}^i) + \delta\Pi_i^*(K_t^*)\right] \quad (33)$$

The main difficulty in solving this dynamic game comes from the discontinuity of the payoff function in K_{t-1}^i. To overcome this point, the proof is divided into two steps: The first step provides a Markov perfect equilibrium of a simplified dynamic game without discontinuity. The second step constructs an equilibrium that is based on the equilibrium of the simplified game.

Step 1 : Solving a simplified dynamic game

Consider a degenerated version of our game in which investment in capacity is fully reversible, i.e. $p^- = p^+$. In this case, the value function is continuous and the strategies function, $K^+(.)$, and the value function $\Pi^+(.)$ should verify that:

$$\begin{aligned} K^{+i}(K_{t-1}) = \arg\max_{K_t^i}\Bigg\{&\left(A + vt - b\sum_{\substack{j=1\\ j\neq i}}^{n} K^{+j}(K_{t-1}) - bK_t^i\right) K_t^i \\ &- p^+(K_t^i - K_{t-1}^i) - \frac{\gamma}{2}(K_t^i - K_{t-1}^i)^2 + \delta\Pi_i^+(K_t)\Bigg\}. \end{aligned} \quad (34)$$

and

$$\Pi_i^+\left(K_{t-1}\right) = \left[\left(A + vt - b\sum_{j=1}^{n} K_t^{+j}\right) K_t^{+i} - p^+(K_t^{+i} - K_{t-1}^i) + \delta\Pi_i^+(K_t^+)\right] \quad (35)$$

The objective of this step is to find two functions $K^+(.)$ and $\Pi^+(.)$ that verify (34) and (35). To do so, we consider a particular value function and determine the strategies $K^+(.)$ that verify (34), then determine that the value function verifies (35).

Step 1.1: definition of the candidate value function

For $i \in \{1, .., n\}$, let us consider the value functions

$$\Pi_i^+(K_t) = \left(\alpha^+ + \mu t - \beta \sum_{j \neq i} K_t^j - \zeta K_t^i\right) K_t^i + Cst^+, \tag{36}$$

for some parameters α^+, μ, β and ζ. The term Cst^+ is a term that does not depend on K_t^i (but may depend on K_t^j for $j \neq i$) and therefore does not affect the determination of the equilibrium: the resolution of (34).

Step 1.2: determination of the equilibrium

The program of firm *i* is now to maximize the function:

$$\begin{aligned}\Lambda^i(K_t^i, K_t^{-i}) = &\left(A + vt - b \sum_{\substack{j=1 \\ j \neq i}}^{n} K_t^j - bK_t^i\right) K_t^i - p^+(K_t^i - K_{t-1}^i) - \frac{\gamma}{2}(K_t^i - K_{t-1}^i)^2 \\ &+ \delta\left(\alpha^+ + \mu t - \beta \sum_{\substack{j=1 \\ j \neq i}}^{n} K_t^j - \zeta K_t^i\right) K_t^i + Cst^+,\end{aligned} \tag{37}$$

as a function of the strategy of its competitor. The game is similar to a classic Cournot game with linear demand and quadratic cost. The best response of firm *i* is given by:

$$K_t^i = \frac{A + vt - b\sum_{j\neq i} K_t^j + \gamma K_{t-1}^i - p^+ + \delta\left(\alpha^+ + \mu t - \beta \sum_{j \neq i} K_t^j\right)}{2b + \gamma + 2\delta\zeta}, \tag{38}$$

and verifies

$$(b + \gamma + 2\delta\zeta - \delta\beta)\, K_t^i = A + vt + \delta\left(\alpha^+ + \mu t\right) - (b + \delta\beta) \sum_{j=1}^{n} K_t^j + \gamma K_{t-1}^i - p^+. \tag{39}$$

(39) is true for all *i*, therefore:

$$(b + \gamma + 2\delta\zeta - \delta\beta) \sum_{j=1}^{n} K_t^j = n\left(A + vt + \delta\left(\alpha^+ + \mu t\right)\right) - n\,(b + \delta\beta) \sum_{j=1}^{n} K_t^j + \gamma \sum_{j=1}^{n} K_{t-1}^j - np^+, \tag{40}$$

which yields:

$$\sum_{j=1}^{n} K_t^j = \frac{n\,(A + vt + \delta\,(\alpha^+ + \mu t)) + \gamma \sum_{j=1}^{n} K_{t-1}^j - np^+}{((n+1)\,b + \gamma + 2\delta\zeta - (n-1)\,\delta\beta)}, \tag{41}$$

and then (39) can be rewritten:

$$K_t^i = \frac{A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\,\gamma\sum_{i=1}^n K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta} \tag{42}$$

with

$$\eta = \left((n+1)\,b + \gamma + 2\delta\zeta + (n-1)\,\delta\beta\,\right). \tag{43}$$

The function $K^+(.)$ defined by:

$$K^+(K_t^i) = \frac{A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\,\gamma\sum_{i=1}^n K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta}, \tag{44}$$

for all $i = 1, .., n$ verifies then (34) for the value functions (34)

Step 1.3: Verifying the dynamic equation

In this sub-step, we verify that the value function given by (36) verifies (35) when the strategies of the firms are given by (44), and determine the values of the parameters α^+, μ, β and ζ of the value function.

(35) can be written as:

$$\begin{aligned}&\left(\left(\alpha^+ + \mu t\right) - \beta\sum_{j\neq i} K_{t-1}^j - \zeta K_{t-1}^i\right) K_{t-1}^i + Cst = -\frac{\gamma}{2}\left(K_{t-1}^i\right)^2 + \delta Cst\\ &+\left(A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+ + \gamma K_{t-1}^i - (b + \delta\beta)\sum_{j\neq i} K_t^{+j} - \left(\frac{\gamma}{2} + \delta\zeta\right) K_t^{+i}\right) K_t^{+i}.\end{aligned} \tag{45}$$

With

$$\sum_{j\neq i} K_t^{+j} = (n-1)\,\frac{A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+}{\eta} + \frac{\eta\gamma\sum_{j\neq i} K_{t-1}^i - (n-1)\,(b + \delta\beta)\,\gamma\sum_{h=1}^n K_{t-1}^h}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta} \tag{46}$$

and

$$K_t^{+i} = \frac{A + vt + \delta\left(\alpha^+ + \mu t\right) - p^+}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\,\gamma\sum_{i=1}^n K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\,\eta}, \tag{47}$$

this can be rewritten as:

$$
\begin{aligned}
&\left(\left(\alpha^{+}+\mu t\right)-\beta \sum_{j \neq i} K_{t-1}^{j}-\zeta K_{t-1}^{i}\right) K_{t-1}^{i}+C s t=-\frac{\gamma}{2}\left(K_{t-1}^{i}\right)^{2}+\delta C s t \\
&-(b+\delta \beta) \frac{\eta \gamma \sum_{j \neq i} K_{t-1}^{i}-(n-1)(b+\delta \beta) \gamma \sum_{h=1}^{n} K_{t-1}^{h}}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta} K_{t}^{* i} \\
&-\left(+\left(\frac{\gamma}{2}+\delta \zeta\right) \frac{\eta \gamma K_{t-1}^{i}-(b+\delta \beta) \gamma \sum_{h=1}^{n} K_{t-1}^{h}}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta}-\gamma K_{t-1}^{i}\right) K_{t}^{* i} \\
&+\left(\eta-(b+\delta \beta)(n-1)-\left(\frac{\gamma}{2}+\delta \zeta\right)\right)\left(\frac{A+v t+\delta\left(\alpha^{+}+\mu t\right)-p^{+}}{\eta}\right) K_{t}^{* i},
\end{aligned} \tag{48}
$$

which yields:

$$
\begin{aligned}
&\left(\left(\alpha^{+}+\mu t\right)-\beta \sum_{j \neq i} K_{t-1}^{j}-\zeta K_{t-1}^{i}\right) K_{t-1}^{i}+C s t=-\frac{\gamma}{2}\left(K_{t-1}^{i}\right)^{2}+\delta C s t \\
&+\left(\frac{\left(b+\frac{\gamma}{2}+\delta \zeta-\delta \beta\right) \eta+(b+\delta \beta)\left(\frac{\gamma}{2}+\delta \zeta+(n-1)(b+\delta \beta)\right)}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta} K_{t-1}^{i}\right) \gamma K_{t}^{* i} \\
&+\left(2 b+\frac{\gamma}{2}+\delta \zeta\right)\left(\frac{A+v t+\delta\left(\alpha^{+}+\mu t\right)-p^{+}}{\eta}\right) K_{t}^{* i} \\
&-\left(2 b+\frac{\gamma}{2}+\delta \zeta\right)\left(\frac{(b+\delta \beta) \sum_{j \neq i} K_{t-1}^{i}}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta}\right) \gamma K_{t}^{* i}
\end{aligned} \tag{49}
$$

$\eta=((n+1) b+\gamma+2 \delta \zeta+(n-1) \delta \beta)$ implies that:

$$
\begin{aligned}
&\left(b+\frac{\gamma}{2}+\delta \zeta-\delta \beta\right) \eta+(b+\delta \beta)\left(\frac{\gamma}{2}+\delta \zeta+(n-1)(b+\delta \beta)\right)=\left(2 b+\frac{\gamma}{2}+\delta \zeta-(b+\delta \beta)\right) \eta \\
&+(b+\delta \beta)\left(\eta-2 b-\frac{\gamma}{2}-\delta \zeta\right)
\end{aligned} \tag{50}
$$

and therefore $\left(b+\frac{\gamma}{2}+\delta \zeta-\delta \beta\right) \eta+(b+\delta \beta)\left(\frac{\gamma}{2}+\delta \zeta+(n-1)(b+\delta \beta)\right)=\left(2 b+\frac{\gamma}{2}+\delta \zeta\right)(\eta-(b+\delta \beta))$. (49) can then be rewritten as:

$$
\begin{aligned}
&\left(\left(\alpha^{+}+\mu t\right)-\beta \sum_{j \neq i} K_{t-1}^{j}-\zeta K_{t-1}^{i}\right) K_{t-1}^{i}+C s t=-\frac{\gamma}{2}\left(K_{t-1}^{i}\right)^{2}+ \\
&\frac{\left(b+\frac{\gamma}{2}+\delta \zeta\right)}{4}\left(\frac{A+v t+\delta\left(\alpha^{+}+\mu t\right)-p^{+}}{\eta}+\frac{(\eta-b-\delta \beta) \gamma K_{t-1}^{i}-(b+\delta \beta) \gamma \sum_{j \neq i} K_{t-1}^{j}}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta}\right)^{2} .
\end{aligned} \tag{51}
$$

As this equation should be true for all K_{t-1}^{j}, $j=1,,n$, and $t \geq 0$, the parameters of the profit function should satisfy:

$$
\alpha^{+}=\frac{1}{2}\left(b+\frac{\gamma}{2}+\delta \zeta\right) \frac{A+\delta \alpha^{+}-p^{+}}{\eta} \frac{(\eta-b-\delta \beta) \gamma}{(b+\gamma+2 \delta \zeta-\delta \beta) \eta} \tag{52}
$$

$$\mu = \frac{1}{2}\left(b+\frac{\gamma}{2}+\delta\zeta\right)\frac{v+\delta\mu}{\eta}\frac{(\eta-b-\delta\beta)\,\gamma}{(b+\gamma+2\delta\zeta-\delta\beta)\,\eta} \tag{53}$$

$$\beta = \frac{1}{2}\left(b+\frac{\gamma}{2}+\delta\zeta\right)\frac{(\eta-b-\delta\beta)\,(b+\delta\beta)\,\gamma^2}{(b+\gamma+2\delta\zeta-\delta\beta)^2\,\eta^2} \tag{54}$$

$$\zeta = -\frac{1}{4}\left(b+\frac{\gamma}{2}+\delta\zeta\right)\frac{(\eta-b-\delta\beta)^2\,\gamma^2}{(b+\gamma+2\delta\zeta-\delta\beta)^2\,\eta^2}+\frac{\gamma}{2}. \tag{55}$$

Equation (52) and (53) give the values of α^+ and μ as functions of ζ and β. More precisely, (52) can be rewritten as:

$$2\,(b+\gamma+2\delta\zeta-\delta\beta)\,\eta^2\alpha^+ = \left(b+\frac{\gamma}{2}+\delta\zeta\right)\left(A+\delta\alpha^+-p^+\right)(\eta-b-\delta\beta)\,\gamma, \tag{56}$$

so

$$\alpha^+ = \frac{\left(b+\frac{\gamma}{2}+\delta\zeta\right)(\eta-b-\delta\beta)\,\gamma\,(A-p^+)}{2\,(b+\gamma+2\delta\zeta-\delta\beta)\,\eta^2-\delta\left(b+\frac{\gamma}{2}+\delta\zeta\right)(\eta-b-\delta\beta)\,\gamma}. \tag{57}$$

(53) can be rewritten as:

$$2\,(b+\gamma+2\delta\zeta-\delta\beta)\,\eta^2\mu = \left(b+\frac{\gamma}{2}+\delta\zeta\right)(\eta-b-\delta\beta)\,\gamma\,(v+\delta\mu)\,, \tag{58}$$

so

$$\mu = \frac{\left(b+\frac{\gamma}{2}+\delta\zeta\right)(\eta-b-\delta\beta)\,\gamma}{2\,(b+\gamma+2\delta\zeta-\delta\beta)\,\eta^2-\delta\left(b+\frac{\gamma}{2}+2\delta\zeta\right)(\eta-b-\delta\beta)\,\gamma}v. \tag{59}$$

With $\eta = ((n+1)\,b+\gamma+2\delta\zeta+(n-1)\,\delta\beta\,)$, (54) can be rewritten as:

$$\zeta = \frac{\gamma}{2}-\frac{1}{2}\frac{(\eta-b-\delta\beta)}{(b+\delta\beta)}\beta. \tag{60}$$

This allows us to determine the value of ζ as a function of β. Indeed, (60) yields:

$$(b+\delta\beta)\,(2\zeta-\gamma) = -\,(\eta-b-\delta\beta)\,\beta, \tag{61}$$

or

$$(b+\delta\beta)\,(2\zeta-\gamma) = -\,(nb+\gamma+2\delta\zeta+(n-2)\,\delta\beta)\,\beta\,, \tag{62}$$

which is equivalent to:

$$2\left(b+2\delta\beta\right)\zeta=\left(b+\delta\beta-\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\beta. \tag{63}$$

This gives us the value of ζ:

$$\zeta=\frac{\left(b+\delta\beta-\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\beta}{2\left(b+2\delta\beta\right)}. \tag{64}$$

β should verify (55), which can be rewritten as:

$$\beta=\frac{1}{2}\left(b+\frac{\gamma}{2}+\delta\zeta\right)\frac{\left(nb+\gamma+2\delta\zeta+\left(n-2\right)\delta\beta\right)\left(b+\delta\beta\right)\gamma^2}{\left(b+\gamma+2\delta\zeta-\delta\beta\right)^2\left(\left(n+1\right)b+\gamma+2\delta\zeta+\left(n-1\right)\delta\beta\right)^2}. \tag{65}$$

This yields:

$$\begin{gathered}4\beta\left(b-\delta\beta+\gamma+2\delta\zeta\right)^2\left(\left(n+1\right)b+\left(n-1\right)\delta\beta+\gamma+2\delta\zeta\right)^2=\\\left(2b+\gamma+2\delta\zeta\right)\left(nb+\gamma+2\delta\zeta+\left(n-2\right)\delta\beta\right)\left(b+\delta\beta\right)\gamma^2.\end{gathered} \tag{66}$$

Using (64), this can written as:

$$\begin{gathered}4\beta\left(\left(b-\delta\beta\right)\left(b+2\delta\beta\right)+\left(1+\delta\right)\left(b+\delta\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\delta\beta\right)^2\left(b+2\delta\beta\right)^2\\\left(\left(\left(n+1\right)b+\left(n-1\right)\delta\beta\right)\left(b+2\delta\beta\right)+\left(1+\delta\right)\left(b+\delta\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\delta\beta\right)^2=\\\left(\left(nb+\left(n-2\right)\delta\beta\right)\left(b+2\delta\beta\right)+\left(1+\delta\right)\left(b+\delta\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\delta\beta\right)\\\left(2b\left(b+2\delta\beta\right)+\left(1+\delta\right)\left(b+\delta\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\delta\beta\right)\left(b+\delta\beta\right)\gamma^2\end{gathered} \tag{67}$$

Any solution of (67) determines some parameter values β, ζ, α^+, μ, and thus ensures that the value function given by (36) verifies (35) when the strategies of the firms are given by (44).

β is defined as a solution of the system:

$$\beta=\frac{1}{2}\left(b+\frac{\gamma}{2}+\delta\zeta\right)\frac{\left(\eta-b-\delta\beta\right)\left(b+\delta\beta\right)\gamma^2}{\left(b+\gamma+2\delta\zeta-\delta\beta\right)^2\eta^2}, \tag{68}$$

where ζ is defined by:

$$\zeta=\frac{\left(b+\delta\beta-\beta\right)\gamma-\left(nb+\left(n-2\right)\delta\beta\right)\beta}{2\left(b+2\delta\beta\right)}. \tag{69}$$

α^+ is defined by:

$$\alpha^+=\frac{\left(b+\frac{\gamma}{2}+\delta\zeta\right)\left(\eta-b-\delta\beta\right)\gamma}{2\left(b+\gamma+2\delta\zeta-\delta\beta\right)\eta^2-\delta\left(b+\frac{\gamma}{2}+\delta\zeta\right)\left(\eta-b-\delta\beta\right)\gamma}\left(A-p^+\right) \tag{70}$$

and μ by:

$$\mu = \frac{\left(b + \frac{\gamma}{2} + 2\delta\zeta\right)(\eta - b - \delta\beta)\gamma}{2(b + \gamma + 2\delta\zeta - \delta\beta)\eta^2 - \delta\left(b + \frac{\gamma}{2} + 2\delta\zeta\right)(\eta - b - \delta\beta)\gamma} v. \tag{71}$$

Step 2 : Solving the main problem

This step is subdivided into three sub-steps: We first construct the strategies and value functions that verify (34) and (35); we then exhibit some properties of the value function; and we finally verify that the constructed strategies and value function verifies (34) and (35).

Step 2.1: construction of the candidate equilibrium strategies and value functions

Let us introduce the parameter α^- defined by:

$$\alpha^- = \frac{\left(b + \frac{\gamma}{2} + \delta\zeta\right)(\eta - b - \delta\beta)\gamma}{2(b + \gamma + 2\delta\zeta - \delta\beta)\eta^2 - \delta\left(b + \frac{\gamma}{2} + \delta\zeta\right)(\eta - b - \delta\beta)\gamma}\left(A - p^-\right). \tag{72}$$

As step 1 is true for all $p^+ > 0$, the strategies function $K^-(.)$ defined by

$$K^{-i}(K_t^i) = \frac{A + vt + \delta(\alpha^- + \mu t) - p^-}{\eta} + \frac{\eta\gamma K_{t-1}^i - (b + \delta\beta)\gamma\sum_{i=1}^n K_{t-1}^j}{(b + \gamma + 2\delta\zeta - \delta\beta)\eta}, \tag{73}$$

for all $i = 1, .., n$ and the value functions:

$$\Pi_i^-(K_t) = \left(\alpha^- + \mu t - \beta\sum_{j\neq i} K_t^j - \zeta K_t^i\right) K_t^i + Cst^-, \tag{74}$$

are solutions to the firm's optimisation problems and the dynamic equations:

$$K^{-i}(K_{t-1}) = \arg\max_{K_t^i}\left\{\left(A + vt - b\sum_{\substack{j=1\\ j\neq i}}^n K^{-j}(K_{t-1}) - bK_t^i\right)K_t^i\right. \tag{75}$$
$$\left. -p^-(K_t^i - K_{t-1}^i) - \frac{\gamma}{2}(K_t^i - K_{t-1}^i)^2 + \delta\Pi_i^-(K_t)\right\},$$

and

$$\Pi_i^-(K_{t-1}) = \left[\left(A + vt - b\sum_{j=1}^n K_t^{-j}\right)K_t^{-i} - C(K_t^{-i} - K_{t-1}^i) + \delta\Pi_i^-(K_t^-)\right]. \tag{76}$$

We have that $K^+(.)$ and $K^-(.)$ are linear, and that, as $p^- < p^+$, $K^+(.) < K^-(.)$.

Let us define $K^*(.)$ as, for $i \in \{1, .., n\}$:

$$K^{*i}(K_{t-1}) = \begin{cases} K^{-*i}(K_{t-1}) \text{ if } K^i_{t-1} > K^{-*i}(K_{t-1}) \\ K^i_{t-1} \text{ if } K^i_{t-1} \geq K^{+*i}(K_{t-1}) \text{ and } K^i_{t-1} \leq K^{-*i}(K_{t-1}) \\ K^{+*i}(K_{t-1}) \text{ if } K^i_{t-1} < K^{+*i}(K_{t-1}) \end{cases} \quad (77)$$

and $K^*_m(.) = K^*(K^*_{m-1}(.))$ with $K^*_1(.) = K^*(.)$. We also define $\Pi^*(.)$ as:

$$\Pi^*(K_0) = \sum_{t=1}^{+\infty} \delta^{t-1} \left(\left(A + vt - b \sum_{j=1}^{n} K^{*j}_t(K_0) \right) K^{*i}_t(K_0) - C\left(K^{*i}_t(K_0) - K^{*i}_{t-1}(K_0) \right) \right). \quad (78)$$

Step 2.2: Properties of the value function

As $\left(A + vt - b \sum_{j=1}^{n} K^{*j}_t(K_0) \right) K^{*i}_t(K_0) - C\left(K^{*i}_t(K_0) - K^{*i}_{t-1}(K_0) \right)$ is finite, $\Pi^*(.)$ uniformly converges. As $K^*_t(.)$ is continuous, $\Pi^*(.)$ is continuous, and as $K^*_t(.)$ is linear, we can show that $\Pi^{*i}(.)$ is concave. First, note that, whatever K_{t-1} we have:

$$C\left(K^{*i}_t(K^i_{t-1}) - K^i_{t-1} \right) - \gamma \left(K^{*i}_t(K^i_{t-1}) - K^i_{t-1} \right)^2 = \begin{cases} p^+ \left(K^{-*i}(K_{t-1}) - K^i_{t-1} \right) \text{ if } K^i_{t-1} > K^{-*i}(K_{t-1}) \\ 0 \text{ if } K^i_{t-1} \geq K^{+*i}(K_{t-1}) \text{ and } K^i_{t-1} \leq K^{-*i}(K_{t-1}) \\ p^- \left(K^{+*i}(K_{t-1}) - K^i_{t-1} \right) \text{ if } K^i_{t-1} < K^{+*i}(K_{t-1}) \end{cases}$$

which is convex. $C\left(K^{*i}_t(K^i_{t-1}) - K^i_{t-1} \right)$ is thus convex.

Let K_0 and K'_0 verify $K^j_0 = K'^j_0$ for all $j \neq i$, and let $\lambda \in [0, 1]$. Then:

$$\begin{aligned} &\Pi^{*i}(\lambda K_0 + (1-\lambda) K'_0) = \\ &\sum_{t=1}^{+\infty} \delta^{t-1} \left(A + vt - b \sum_{j=1}^{n} \left(\lambda K^{*j}_t(K_0) + (1-\lambda) K^{*j}_t(K'_0) \right) \right) * \left(\lambda K^{*j}_t(K_0) + (1-\lambda) K^{*j}_t(K'_0) \right) \\ &- C\left(\lambda K^{*i}_t(K_0) + (1-\lambda) K^{*i}_t(K'_0) - \lambda K^{*j}_{t-1}(K_0) - (1-\lambda) K^{*j}_{t-1}(K'_0) \right) \end{aligned} \quad (79)$$

By convexity of the investment cost, we have:

$$\begin{aligned} \Pi^*(\lambda K_0 + (1-\lambda) K'_0) \geq &\sum_{t=1}^{+\infty} \delta^{t-1} \left(A + vt - b \sum_{j=1}^{n} \lambda K^{*j}_t(K_0) \right) \lambda K^{*j}_t(K_0) \\ &+ \left(A + vt - b \sum_{j=1}^{n} (1-\lambda) K^{*j}_t(K'_0) \right) \lambda K^{*j}_t(K_0) - \lambda C\left(K^{*i}_t(K_0) - K^{*j}_{t-1}(K_0) \right) \\ &+ (1-\lambda) C\left(K^{*i}_t(K'_0) - K^{*j}_{t-1}(K'_0) \right) \end{aligned} \quad (80)$$

Π^{*i} is therefore concave. Let us define ξ^+ as:

$$\xi^+ = \inf_K \left\{ K^{+i}(K) = K \text{ and } K^i = K^j \text{ for all } i, j = 1, .., n \right\}, \quad (81)$$

and ξ^- as:

$$\xi^- = \inf_{K \text{ symmetric}} \left\{ K^{-i}(K) = K \text{ and } K^i = K^j \text{ for all } i,j = 1,..,n \right\}. \quad (82)$$

The strategies given by (77) can then be rewritten:

$$K^{*i}(K_{t-1}) = \begin{cases} K^{-*i}(K_{t-1}) \text{ if } K^i_{t-1} > \xi^- \\ K^i_{t-1} \text{ if } K^i_{t-1} \in [\xi^+, \xi^-] \\ K^{+*i}(K_{t-1}) \text{ if } K^i_{t-1} < \xi^+ \end{cases}$$

as $\Pi^*(.)$ uniformly converges, and the strategies $K^{*i}(.)$ are differentiable everywhere except in ξ^+ and ξ^-, $\Pi^*(.)$ is differentiable everywhere except in ξ^+ and ξ^- . ξ^+ and ξ^- are given by:

$$\xi^+ = \frac{A + vt + \delta(\alpha^+ + \mu t) - p^+}{(\eta - \gamma)}, \quad (83)$$

and

$$\xi^- = \frac{A + vt + \delta(\alpha^- + \mu t) - p^-}{(\eta - \gamma)}. \quad (84)$$

Step 2.3: the candidate strategies form a Markov perfect equilibrium
By construction, $\Pi^*(.)$ verifies (35) for $K^*(.)$ defined in (77).

Assume now that $K^i_{t-1} = K^j_{t-1}$ for all $i,j = 1,..,n$. When $K^i_{t-1} < \xi^+$ for all i, then $\Pi^*(.)$ as defined by (78) verifies $\Pi^*(.) = \Pi^+(.)$. In that case, $K^+(K_{t-1})$, which verifies (34), also verifies (32) ($K^{+i}(K_{t-1})$ is a local maximum for each $i = .., n$, and, as $\Pi^*(.)$ is concave, $K^{+i}(K_{t-1})$ is also the global maximum). With the same reasoning, when $K^i_{t-1} > \xi^-$ for all i, $\Pi^*(.)$ as defined by (78) verifies $\Pi^*(.) = \Pi^+(.)$ and $K^-(K_{t-1})$ also verifies (32) as it verifies (34).

To see what happen when $K^i_{t-1} \in [\xi^+, \xi^-]$, let us define $\Lambda(.,.)$, the objective of firm i when $K^j_t = K^j_{t-1}$ for $j \neq i$:

$$\Lambda\left(K^i_t, K^i_{t-1}\right) = \left(A + vt - b \left(K^i_t + \sum_{\substack{j=1 \\ j \neq i}}^{n} K^{*j}(K_{t-1}) \right) \right) K^i_t - C(K^i_t - K^i_{t-1}) + \delta \Pi^{*i}(K_t). \quad (85)$$

As $K^{*j}(.)$ is differentiable everywhere except in ξ^+ and ξ^-, we can then define the left-hand and right-hand differentiable of Λ in every K^i_{t-1} except ξ^+ and ξ^-:

$$\frac{\partial^+ \Lambda}{\partial K^i_t}(K^i_{t-1}) = \lim_{h \to 0} \frac{\partial \Lambda}{\partial K^i_t}\left(K^i_{t-1} + h, K^i_{t-1}\right) \quad (86)$$

$$\frac{\partial^+ \Lambda}{\partial K_t^i}(K_{t-1}^i) = \left(A + vt - b \left(2K_t^i + \sum_{\substack{j=1 \\ j \neq i}}^{n} K^{*j}(K_{t-1}) \right) \right) - p^+ + \delta \frac{\partial \Pi^{*i}(K_t)}{\partial K_t^i}, \quad (87)$$

and

$$\frac{\partial^- \Lambda}{\partial K_t^i}(K_{t-1}^i) = \lim_{h \to 0} \frac{\partial \Lambda}{\partial K_t^i}\left(K_{t-1}^i - h, K_{t-1}^i\right) \quad (88)$$

$$\frac{\partial^- \Lambda}{\partial K_t^i}(K_{t-1}^i) = \left(A + vt - b \left(2K_t^i + \sum_{\substack{j=1 \\ j \neq i}}^{n} K^{*j}(K_{t-1}) \right) \right) - p^- + \delta \frac{\partial \Pi^{*i}(K_t)}{\partial K_t^i} \quad (89)$$

As $\Pi^{*i}(.)$ is concave, $\frac{\partial^+ \Lambda}{\partial K_t^i}(.)$ and $\frac{\partial^- \Lambda}{\partial K_t^i}(.)$ are decreasing. As $p^+ > p^-$, $\frac{\partial^+ \Lambda}{\partial K_t^i}(.) < \frac{\partial^- \Lambda}{\partial K_t^i}(.)$.

Furthermore, as ξ^+ is the first fixed point of $K^+(.)$ and $K^*(.)$ maximize (32) when $K_{t-1}^i < \xi^+$ we know that:

$$\lim_{\substack{K_{t-1}^i \to \xi^+ \\ K_{t-1}^i < \xi^+}} \frac{\partial^+ \Lambda}{\partial K_t^i}(K_{t-1}^i) = 0, \quad (90)$$

and, similarly:

$$\lim_{\substack{K_{t-1}^i \to \xi^- \\ K_{t-1}^i > \xi^-}} \frac{\partial^- \Lambda}{\partial K_t^i}(K_{t-1}^i) = 0. \quad (91)$$

Assume now that $K_{t-1}^i \in [\xi^+, \xi^-]$. As $\frac{\partial^+ \Lambda}{\partial K_t^i}(.)$ is decreasing, (90) implies that:

$$\lim_{\substack{K_{t-1}^i \to \xi^+ \\ K_{t-1}^i > \xi^+}} \frac{\partial^+ \Lambda}{\partial K_t^i}(K_{t-1}^i) \leq 0, \quad (92)$$

and (91) implies that:

$$\lim_{\substack{K_{t-1}^i \to \xi^+ \\ K_{t-1}^i < \xi^+}} \frac{\partial^+ \Lambda}{\partial K_t^i}(K_{t-1}^i) \geq 0. \quad (93)$$

Then, for every $K_{t-1}^i \in]\xi^+, \xi^-[$, as $\frac{\partial^+ \Lambda}{\partial K_t^i}(.) < \frac{\partial^- \Lambda}{\partial K_t^i}(.)$, $\frac{\partial^+ \Lambda}{\partial K_t^i}(K_{t-1}^i) < 0$ and $\frac{\partial^- \Lambda}{\partial K_t^i}(K_{t-1}^i) > 0$ then firm $i's$ optimal choice of capacity is $K_t^i = K_{t-1}^i$. ■

Appendix B

To control for the existence of the two transition effects and the lingering effect, the following tables show the average over 1000 simulations of:

In column *T*1, the difference between the price in the first period after the cartel starts and at end of the cartel (it should be null if the first transition effect is not present), for initial capacities such that their marginal profitability equals the price of investment;

In column *T*2, the difference between the price one period after the end of the cartel and 10 periods later (it should be null if the second transition effect is not present), for initial capacities such that their marginal profitability equals the price of investment;

In column *T*3, the difference between the price at the beginning of the simulation and the price at the end of the simulation and therefore after the collusion interlude (it should be nul if the lingering effect is not present), for initial capacities such that their marginal profitability equals the cost of disinvestment.

The first column gives the value of the parameter that varies compared to our main scenario.

Funding Open access funding provided by Stellenbosch University.

References

Back, K., & Paulsen, D. (2009). Open-loop equilibria and perfect competition in option exercise games. *Review of Financial Studies, 22*(11), 4531–4552.

Besanko, D., & Doraszelski, U. (2004). Capacity dynamics and endogenous asymmetries in firm size. *RAND Journal of Economics*. https://doi.org/10.2307/1593728

Boshoff, W. H., & Van Jaarsveld, R. (2019). Recurrent collusion: Cartel episodes and overcharges in the South African cement market. *Review of Industrial Organization, 54*, 353–380.

Boswijk, H. P., Bun, M. J., & Schinkel, M. P. (2019). Cartel dating. *Journal of Applied Econometrics, 34*(1), 26–42.

Boyer, M., Lasserre, P., & Moreaux, M. (2012). A dynamic duopoly investment game without commitment under uncertain market expansion. *International Journal of Industrial Organization, 30*(6), 663–681.

Case 1:08-cv-05214, Judge James B. Zagel, 2015, Memorandum Opinion and Order - Steel Antitrust Litigation Settlement.

Davis, P., & Garcé s, E. (2009). Quantitative techniques for competition and antitrust analysis. Princeton University Press

Fagart, T. (2022). Collusion in capacity under irreversible investment. *International Journal of Industrial Organization, 81*, Article 102810.

Feuerstein, S., & Gersbach, H. (2003). Is capital a collusion device? *Economic Theory, 21*(1), 133–154.

Figuières C. (2009). Markov interactions in a class of dynamic games. *Theory and Decision,66*, 39–68.

Frank, N., & Schliffke, P. (2013). The post-cartel equilibrium puzzle in the German cement market: A reply to H schelrath, M ller, and Veith. *Journal of Competition Law & Economics, 9*(2), 495–509.

Genc, T., & Zaccour, G. (2014). Capacity Investments in a Stochastic Dynamic Game: Good News Principle. *Operations Research Letters, 41*(5), 482–485.

Grenadier, S. (2002). Option exercise games: An application to the equilibrium investment strategies of firms. *Review of Financial Studies, 15*(3), 691–721.

Hü schelrath, K., M ller, K., & Veith, T. (2013). Concrete shoes for competition: The effect of the German cement cartel on market price. Journal of Competition Law and Economics,9(1), 97–123

Harrington, J. E. (2004). Post-cartel pricing during litigation. *Journal of Industrial Economics, 52*, 517–533.

Huisman, K., & Kort, P. (2015). Strategic capacity investment under uncertainty. *The RAND Journal of Economics, 46*(2), 376–408.

Paha, J. (2011). Empirical methods in the analysis of collusion. *Empirica, 38*, 389–415.

Paha, J. (2017). The Value of Collusion with Endogenous Capacity and Demand Uncertainty. *The Journal of Industrial Economics, 65*(3), 623–653.

Porter, R. H. (1983). A study of cartel stability: The Joint Executive Committee, 1880-1886. *The Bell Journal of Economics*. https://doi.org/10.2307/3003634

Reynolds, S. (1991). Dynamic oligopoly with capacity adjustment costs. *Journal of Economic Dynamics and Control, 15*, 491–514.

Rubinfeld, D. L. (1985). Econometrics in the Courtroom. *Colum. L. Rev., 85*, 1048.

Rubinfeld, D. L. (2012). Antitrust damages. *Research handbook on the economics of antitrust law.* Edward Elgar Publishing.

White, H., Marshall, R., & Kennedy, P. (2006). The measurement of economic damages in antitrust civil litigation. *Econ. Comm. Newsl., 6*, 17.